If ${}^{x}C_2 = 6$, Find the value of x.

Solution

$$\frac{\underline{|x}}{\underline{|2}\ \underline{|x-2}} = 6 \Rightarrow \frac{x(x-1)\cancel{\underline{|x-2}}}{2\ \cancel{\underline{|x-2}}} = 6$$

$\therefore x(x-2) = 12$ *OR* $x = 4$

If $\underline{|n} = 720,\ {}^{n+1}C_r : {}^{n+1}C_{r-1} = 3:5$, Find the values of n and r.

Solution

Step 1: since $720 = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = \underline{|6}\ \therefore n = 6$

Step 2: since ${}^{7}C_r : {}^{7}C_{r-1} = 3:5$

$\therefore (7 - r + 1):r = 3:5 \Rightarrow \dfrac{8-r}{r} = \dfrac{3}{5}$

$\therefore 3r = 40 - 5r \Rightarrow 8r = 40 \Rightarrow r = 5$

Corollary :

Let $x \in R^+, n \in Z^+$

Then

$(1 + x)^n = 1 + nx + {}^nC_2x^2 + \ldots\ldots + {}^nC_rx^r + \ldots\ldots + x^n$

And

$(1 - x)^n = 1 - nx + {}^nC_2x^2 + \ldots\ldots + {}^nC_r(-x)^r + \ldots + (-x)^n$

The general term in the expansion of $(x + a)^n$ is T_{r+1}, where :

$$T_{r+1} = {}^nC_r\, a^r x^{r-1}$$

$$(a+b)^n = a^n + na^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots\dots + \boxed{{}^nC_r a^{n-r} b^r} + \dots\dots + nab^{n-1} + b^n$$

The middle term or the middle two terms of the expansion of $(a + b)^n$

Remember that the number of terms = n + 1

- **If n is even, the number of terms will be odd, and the middle term is: $T_{\frac{n}{2}+1}$**
- **If n is odd, the number of terms will be even, and the two middle terms are: $T_{\frac{n+1}{2}}$ and $T_{\frac{n+3}{2}}$**

$$\boxed{\frac{T_{r+1}}{T_r} = \frac{n - r + 1}{r} \times \frac{b}{a}}$$

Find the term free of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^9$

Solution

Since $T_{r+1} = {}^9C_r\,(x^2)^{9-r}\left(\frac{1}{x}\right)^r$

$\therefore T_{r+1} = {}^9C_r\,\frac{x^{18-2r}}{x^r} = {}^9C_r\,x^{18-3r}$

according to the required,

$0 = 18 - 3r \Rightarrow 3r = 18 \Rightarrow r = 6$

replacing r by 6, we get : $T_7 = {}^9C_6 = 84$

Evaluate the coefficient of x^6 in the expansion of $(2 - x^2)^5$

Solution

Since $T_{r+1} = {}^5C_r\,(2)^{5-r}\,(-x^2)^r$

$\therefore T_{r+1} = (-1)^r\,{}^5C_r\,(2)^{5-r}\,x^{2r}$

according to the required,

we put $2r = 6$ & so $r = 3$

replacing r by 3, we get :

$T_4 = -{}^5C_3(2)^2x^6 \Rightarrow T_4 = -40x^6$

Therefore the required coefficient is (- 40)

If $T_3 = 112$, $T_4 = 448$ and $T_5 = 1120$ in the expansion of $(a + b)^n$, find the value of n.

Solution

$\because \frac{T_4}{T_3} = \frac{n-3+1}{3} \times \frac{b}{a}$ $\quad \therefore \frac{n-2}{3} \times \frac{b}{a} = \frac{448}{112} = 4$ $\quad \therefore \frac{b}{a} = \frac{12}{n-2}$ *(i)*

$\because \frac{T_5}{T_4} = \frac{n-4+1}{4} \times \frac{b}{a}$ $\quad \therefore \frac{n-3}{4} \times \frac{b}{a} = \frac{1120}{448} = \frac{5}{2}$ $\quad \therefore \frac{b}{a} = \frac{10}{n-3}$ *(ii)*

(i) & (ii) $\Rightarrow \frac{12}{n-2} = \frac{10}{n-3}$

$6n - 18 = 5n - 10 \Rightarrow n = 8$

Put the complex number $z = -1 + \sqrt{3}\ i$ in the trigonometric form.

Solution $r = \sqrt{(-1)^2 + \sqrt{3}^{\,2}} = 2$

Since $\cos\theta = -\frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$

And Since $\cos\theta$ is negative and $\sin\theta$ is positive,

therefore θ lies in the second quadrant.

therefore $\theta = 180^\circ - 60^\circ = 120^\circ = \frac{2\pi}{3}$ rad

z in the trigonometric form $= 2(\cos 120^\circ + i \sin 120^\circ)$

$= 2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$

Put the complex number $z = -1 - \sqrt{3}\, i$ in the trigonometric form.

Solution $r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

Since $\cos\theta = -\frac{1}{2}$ and $\sin\theta = -\frac{\sqrt{3}}{2}$

And Since both $\sin\theta$ and $\cos\theta$ are negative,

therefore θ lies in the third quadrant.

therefore $\theta = 180^\circ + 60^\circ = 240^\circ = \frac{4\pi}{3}$ rad

z in the trigonometric form $= 2(\cos 240^\circ + i \sin 240^\circ)$

$$= 2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$$

If $z_1 = r_1(\cos\theta_1 + i \sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i \sin\theta_2)$

then

$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$

Raising a Complex number to a positive integer power

If $z = r(\cos\theta + i \sin\theta)$

then $z^n = r^n(\cos n\theta + i \sin n\theta)$ where $n \in Z^+$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$$

De Moivre theorem (without proof)

$$(\cos\theta + i \sin\theta)^{\frac{1}{k}} = \cos\frac{\theta + 2\pi n}{k} + i \sin\frac{\theta + 2\pi n}{k}$$

where n takes the values $1, 2, \dots, k - 1$

Put the complex number $z = 1 - \sqrt{3}\ i$ in the trigonometric form.

Solution $r = \sqrt{(-1)^2 + \sqrt{3}^{\,2}} = 2$

Since $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$

And Since $\cos\theta$ is positive and $\sin\theta$ is negative,

therefore θ lies in the fourth quadrant.

therefore $\theta = 360^\circ + 60^\circ = 300^\circ = \frac{5\pi}{3}$ rad

z in the trigonometric form $= 2(\cos 300^\circ + i \sin 300^\circ)$

$= 2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})$

$z = re^{\theta i}$ is called the exponential form of the complex number $z = x + iy$

Put the complex number $z = 1 + \sqrt{3}\ i$ in its exponential form.

Solution

$r = \sqrt{1^2 + \sqrt{3}^{\,2}} = 2$

Since $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$

And Since both $\sin\theta$ and $\cos\theta$ are positive,

therefore θ lies in the first quadrant.

therefore $\theta = 60^\circ = \frac{\pi}{3}$ rad

z in the exponential form $= 2\, e^{\frac{\pi}{3} i}$

The sum of the cubic roots of 1 = 0
i.e. $1 + w + w^2 = 0$

The product of the cubic roots of 1 = 1
i.e. $w^3 = 1$

Since $1 + w + w^2 = 0$ *Therefore*

1- $1 + w = -w^2$
2- $1 + w^2 = -w$
3- $w + w^2 = -1$

Since $1 \times w \times w^2 = 1$ *Therefore*

1- $\frac{1}{w} = w^2$
2- $\frac{1}{w^2} = w$
3- $w^{3n} = 1$
4- $w^{3n+1} = w$
5- $w^{3n+2} = w^2$ where $n \in Z+$

Prove that: $(w + \frac{1}{w}) + (w^2 - \frac{1}{w}) = -2$

Solution

$L.H.S = (w + w^2)^2 + (w^2 - w)^2$

$= (-1)^2 + w^4 - 2w^3 + w^2$

$= 1 + w + w^2 - 2 = -2$

Prove that : $\frac{a + wb}{w^2a + b} + \frac{cw^2 + d}{c + dw} = -1$

Solution

$W^3 = 1$

$$L.H.S = \frac{w^3a + wb}{w^2a + b} + \frac{cw^2 + dw^3}{c + dw}$$

$$\therefore L.H.S = \frac{w\cancel{(w^2a + b)}}{\cancel{w^2a + b}} + \frac{w^2\cancel{(c + dw)}}{\cancel{c + dw}}$$

$\therefore L.H.S = w + w^2 = -1$

Prove that : $(2 + 7w + 2w^2)(2 + 7w^2 + 2w) = 25$

Solution

Let $7w = 2w + 5w$ and $7w^2 = 2w^2 + 5w^2$

$L.H.S = (2 + 2w + 2w^2 + 5w)(2 + 2w + 2w^2 + 5w^2)$

Since $2 + 2w + 2w^2 = 0$

$\therefore L.H.S = (5w)(5w^2) = 25w^3 = 25$

Properties of determinants

Property 1

The value of a determinant is unchanged if its rows and columns are exchanged in the same order.

$$\begin{vmatrix} 10 & 5 & a \\ 11 & 3 & b \\ -1 & 2 & c \end{vmatrix} = \begin{vmatrix} 10 & 11 & -1 \\ 5 & 3 & 2 \\ a & b & c \end{vmatrix}$$

Property 2

The sign of a determinant changes if we interchange two rows or two columns.

$$\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} = - \begin{vmatrix} 2 & 5 & 8 \\ 1 & 4 & 7 \\ 3 & 6 & 9 \end{vmatrix}$$

Property 3

If two rows or two columns of a determinant are identical, the value of the determinant is equal to zero

$$\begin{vmatrix} 1 & 4 & 7 \\ 1 & 4 & 7 \\ 3 & 6 & 9 \end{vmatrix} = 0 \ \& \ \begin{vmatrix} 5 & 5 & 1 \\ 8 & 8 & 2 \\ 4 & 4 & 3 \end{vmatrix} = 0$$

Property 4

If the elements of a row or a column of a determinant are multiplied by any number m, the determinant is multiplied by the same number m.

$$\begin{vmatrix} 2 & 3 & 4 \\ 15 & -5 & 10 \\ 1 & 4 & -3 \end{vmatrix} = \boxed{5} \begin{vmatrix} 2 & 3 & 4 \\ 3 & -1 & 2 \\ 1 & 4 & -3 \end{vmatrix}$$

$$\begin{vmatrix} 10 & 3 & 4 \\ 15 & -1 & 2 \\ 5 & 4 & -3 \end{vmatrix} = \boxed{5} \begin{vmatrix} 2 & 3 & 4 \\ 3 & -1 & 2 \\ 1 & 4 & -3 \end{vmatrix}$$

Property 5

If the elements of a row or a column of a determinant are zeros, then the value of the determinant is equal to zero.

$$\begin{vmatrix} 0 & 0 & 0 \\ 3 & -1 & 2 \\ 1 & 4 & -3 \end{vmatrix} = 0 \ \& \ \begin{vmatrix} 0 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 4 & -3 \end{vmatrix} = 0$$

Property 6

if each of the elements of a row or a column of a determinant is multiplied by the cofactor of the corresponding element of another row or column, the sum of the products is equal to zero.

$$\begin{vmatrix} a & 0 & 0 \\ d & b & 0 \\ e & f & c \end{vmatrix} = abc \ \& \ \begin{vmatrix} 0 & 0 & a \\ 0 & b & d \\ c & f & e \end{vmatrix} = -abc$$

Solved problem no. 1

Prove that : $\begin{vmatrix} 1 & a & a \\ 1 & b & a \\ 1 & a & -b \end{vmatrix} = (a+b)(a-b).$

Solution

$$\Delta = \begin{vmatrix} 1 & a & a \\ 1 & b & a \\ 1 & a & -b \end{vmatrix} \xrightarrow[r_2 - r_3]{r_1 - r_2 \ \&}$$

$$\therefore \Delta = \begin{vmatrix} 0 & a-b & o \\ 0 & b-a & a+b \\ 1 & a & -b \end{vmatrix} \xrightarrow{r_1 + r_2}$$

$$\therefore \Delta = \begin{vmatrix} 0 & 0 & a+b \\ 0 & b-a & a+b \\ 1 & a & -b \end{vmatrix} = -(a+b)(b-a)$$

$$\therefore \Delta = (a+b)(a-b).$$

Solved problem no. 2

If $(x-1)$ is a factor of the determinant $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & 1 & x+1 \\ -1 & 1 & x+k \end{vmatrix}$ find the value of k.

Solution

since $(x-1)$ is a factor of the determinant, therefore the determinant= 0 as $x = 1$

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 1+k \end{vmatrix} = 0 \xrightarrow{r_2 + r_3}$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 3+k \\ -1 & 1 & 1+k \end{vmatrix} = 0 \xrightarrow{c_2 - c_3}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1-k & 3+k \\ -1 & -k & 1+k \end{vmatrix} = 0$$

$$\therefore -k - 1 = 0 \text{ or } k = -1$$

Solved problem no. 1

Use Cramer's method to find the S.S. of the equations:
$x - y = -2$ and $2x + 3y = 1$

Solution

$$\Delta = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = (1 \times 3) - (-1 \times 2) = 5,$$

$$\Delta_x = \begin{vmatrix} -2 & -1 \\ 1 & 3 \end{vmatrix} = (-2 \times 3) - (-1 \times 1) = -5 \quad \& \quad \Delta_y = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = (1 \times 1) - (-2 \times 2) = 5$$

$$x = \frac{-5}{5} = -1 \;\&\; y = \frac{5}{5} = 1 \quad \textit{and so } S.S = \{(-1, 1)\}$$

Solved problem no. 2

Use Cramer's method to find the S.S. of the equations;
$x + y + z = 3$, $x - y + z = 1$ and $x + y - 2z = 0$.

Solution

$$\Delta = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -2 & 1 & 1 \end{array}\right| \qquad \Delta_x = \left|\begin{array}{ccc|cc} 3 & 1 & 1 & 3 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 0 & 1 & -2 & 0 & 1 \end{array}\right|$$

$$\Delta = (2 + 1 + 1) - (-1 + 1 - 2) = 6 \qquad \Delta_x = (6 + 0 + 1) - (0 + 3 - 2) = 6$$

$$\Delta_y = \left|\begin{array}{ccc|cc} 1 & 3 & 1 & 1 & 3 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -2 & 1 & 0 \end{array}\right| \qquad \Delta_z = \left|\begin{array}{ccc|cc} 1 & 1 & 3 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right|$$

$$\Delta_y = (-2 + 3 + 0) - (1 + 0 - 6) = 6 \qquad \Delta_z = (0 + 1 + 3) - (-3 + 1 + 0) = 6$$

$$x = y = z = \frac{6}{6} = 1 \quad \textit{and so } S.S = \{(1, 1, 1)\}$$

If two planes have three distinct non-collinear points in common , then they coincide .

The angle between two skew lines is the angle between two intersecting lines respectively parallel to the given skew lines

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1. The Right Pyramid :

It is the pyramid whose base is a regular polygon & its altitude (height) passes through the centre of its base.

2 - The Regular Pyramid:

It is a triangular pyramid whose base & faces are all equilateral triangles.

If a line is parallel to a plane , then it is parallel to every line of intersection of this plane with the planes containing the given line .

If a line is not a subset of a plane is parallel to a line in the plane , then it is parallel to the plane .

If a plane intersects two parallel planes , then its lines of intersection with these planes are parallel .

If a line intersects one of two parallel planes , then it intersct the other .

If two lines are parallel , & contained into two different intersecting planes , then the two lines are parallel to the line of intersection of the two planes .

If a line is parallel to two intersecting planes , then it is parallel to their line of intersection.

If two intersecting lines are respectively parallel to another two intersecting lines, then the two planes containing each pair of intersecting lines are parallel .

If two straight lines intersect a set of parallel planes , then the lengths of the line segments intercepted between these planes are proportional.

A line is said to be perpendicular to a plane if it is perpendicular to every line in the plane.

If a line is perpendicular to two intersecting lines at their point of intersection , then it is perpendicular to their plane.

1. If a line is perpendicular to two intersecting lines , then it is perpendicular to their plane .

2. All perpendiculars to a line at a point on it lie in a unique plane .

3. There is one and only one plane perpendicular to a line at a point on it .

4. If two lines are perpendicular to a plane , they are parallel to each other.

5. If a line is perpendicular to two planes , then the two planes are parallel .

The angle between a line and a plane is the angle between any line segment of the line and its projection on the plane.

If a line inclined to a plane is perpendicular to a line in the plane , then its projection on the plane is perpendicular to the line in the plane.

If the projection of a line inclined to a plane is perpendicular to a line in the plane, then the inclined line is perpendicular to the line in the plane.

A dihedral angle

is a convex figure bounded by half-planes X and Y emanating from one straight line $\overleftrightarrow{AB} = X \cap Y$.

The plane angle of a dihedral angle

is the angle formed by the lines of intersection of the two planes with a plane perpendicular to its edges.

Two planes X and Y are perpendicular to one another if any of the resulting 4 dihedral angles is a right angle.

If a line is perpendicular to a plane , then every plane containing this line is perpendicular to this plane.

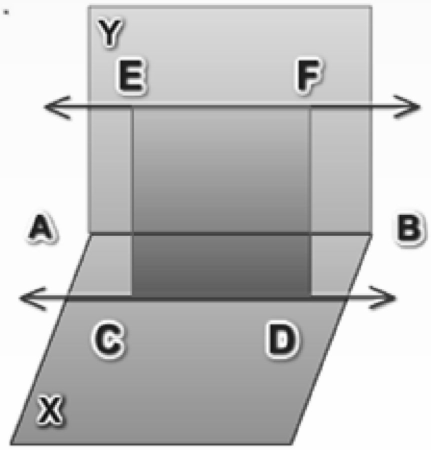
If a line contained in one of two perpendicular planes is perpendicular to their line of intersection , then this line is perpendicular to the other plane .

If two planes are individually perpendicular to a 3rd plane, then their line of intersection is perpendicular to the 3rd plane .

The angle between two skew lines is the angle between two intersecting lines respectively parallel to the given skew lines

1. X & Y are two intersecting planes at $\overleftrightarrow{AB}$.
Draw $\overleftrightarrow{CD}$ in X such that $\overleftrightarrow{CD}$ // Y ,
Draw $\overleftrightarrow{EF}$ in Y such that $\overleftrightarrow{EF}$ // X .

Prove that : (*I*) $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$ (*II*) $\overleftrightarrow{AB}$ // the plane EFDC .

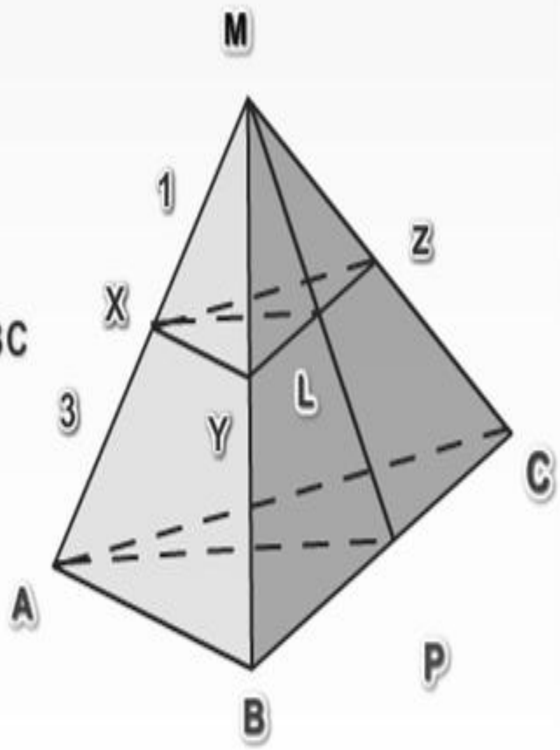


Solution

Since $\overleftrightarrow{CD} \subset$ X and $\overleftrightarrow{CD}$ // Y, therefore $\overleftrightarrow{CD}$ // $\overleftrightarrow{AB}$ ... (1)
Since $\overleftrightarrow{EF} \subset$ Y and $\overleftrightarrow{EF}$ // X, therefore $\overleftrightarrow{EF}$ // $\overleftrightarrow{AB}$ ...(2)
From (1) & (2) we deduce that $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$ (R.T.P. 1)
Since $\overleftrightarrow{EF}$ // $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF} \subset$ plane EFDC
Therefore $\overleftrightarrow{AB}$ // the plane EFDC

In the given figure, MABC is a triangular pyramid.
X , Y & Z are drawn on $\overline{MA}$, $\overline{MB}$ & $\overline{MC}$ respectively such that MX : XA = MY : YB = MZ : ZC = 1 : 3 .

1- Prove that plane XYZ // plane ABC .
2- If point P is drawn on $\overline{BC}$ and $\overline{MP}$ is drawn to cut $\overline{YZ}$ at L , prove that AP = 4 XL.



in Δ MAB :

Since $\frac{MX}{XA} = \frac{MY}{YB}$ Therefore $\overline{XY}$ // $\overline{AB}$

similarly : $\overline{YZ}$ // $\overline{BC}$

Since $\overline{XY}$ and $\overline{YZ}$ plane XYZ, $\overline{AB}$ and $\overline{BC}$ plane ABC

Therefore plane XYZ // plane ABC (R.T.P.1)

Since plane XYZ // plane ABC, $\overline{XL} \subset$ plane XYZ,

$\overline{AP} \subset$ plane ABC Therefore $\overline{XL}$ // $\overline{AP}$

in Δ MAP :

Since $\overline{XL}$ // $\overline{AP}$ Therefore $\frac{MX}{MA} = \frac{XL}{AB} = \frac{1}{4}$

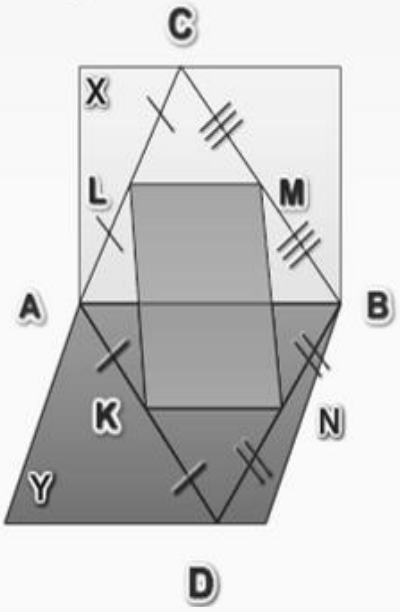
Therefore AP = 4 XL (R.T.P.2)

In the given figure, CAB and DAB are two triangles drawn in two different planes. L, M, K, N are the mid-points of $\overline{CA}$, $\overline{CB}$, $\overline{DA}$, $\overline{DB}$ respectively.

Prove that :

i) $\overline{LM} \parallel \overline{KN}$

ii) $\overline{AB} \parallel$ plane LMNK



In the given figure, $\overline{AB}$ is a diameter of a circle, C is a point on arc AB.

Draw $\overleftrightarrow{AD}$ perpendicular to the plane of the circle.

since $\overline{AB}$ is a diameter, therefore $\overline{BC} \perp \overline{AC}$ (I)

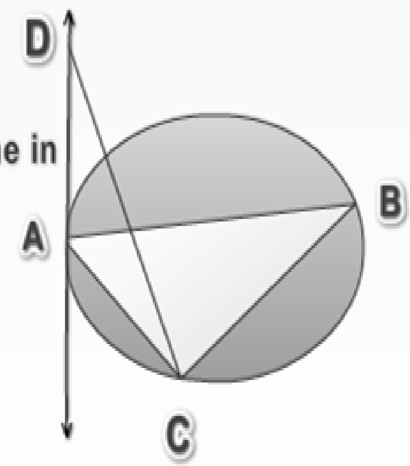
& since $\overleftrightarrow{AD} \perp$ the plane of the circle, therefore $\overleftrightarrow{AD} \perp$ any line in the plane of the circle.

as a result : $\overline{BC} \perp \overleftrightarrow{AD}$ (II)

From (I) & (II)

we conclude that $\overline{BC} \perp$ any line in the plane ACD,

which implies $\overline{BC} \perp \overline{CD}$ (Q.E.D).



In the given figure, let the dimensions of the cuboid l, w and h and let the length of its diagonal d.

Prove that $d^2 = l^2 + w^2 + h^2$

Since $\overline{DC} \perp$ both $\overline{BC}$ and $\overline{CE}$,

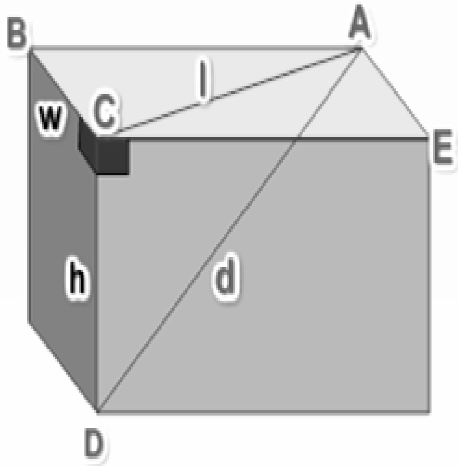
therefore $\overline{DC} \perp$ plane ABCD

Therefore $\overline{DC} \perp \overline{AC}$

In ΔACD right angled at C, $d^2 = AC^2 + h^2$ ------ (*I*)

But in ΔABC right angled at B, $AC^2 = l^2 + w^2$

by substitution in (*I*) we get $d^2 = l^2 + w^2 + h^2$



In any regular pyramid of edge length l and height h.
prove that $2l^2 = 3h^2$

Solution

Since ABC is equilateral Δ

Therefore CD = CB sin 60° $= l \dfrac{\sqrt{3}}{2}$

Since $\overline{CD}$ is a median in Δ ABC

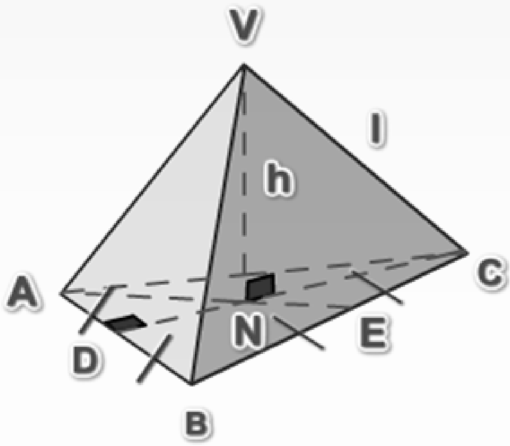
Therefore CN $= \dfrac{2}{3}$ CD $= \dfrac{2}{3} \left(l \dfrac{\sqrt{3}}{2}\right)$

Therefore $(CN)^2 = \dfrac{l^2}{3}$

In the right angled triangle Δ VNC, right angled at N,

$(VC)^2 = (CN)^2 + (VN)^2$

i.e. $l^2 = \dfrac{l^2}{3} + h^2$ or $\dfrac{2l^2}{3} = h^2$ or $2l^2 = 3h^2$



ABCD is a triangular pyramid in which $\overline{AB} \perp \overline{CD}$.

Draw $\overline{AH} \perp \overline{CD}$ to cut it at H.

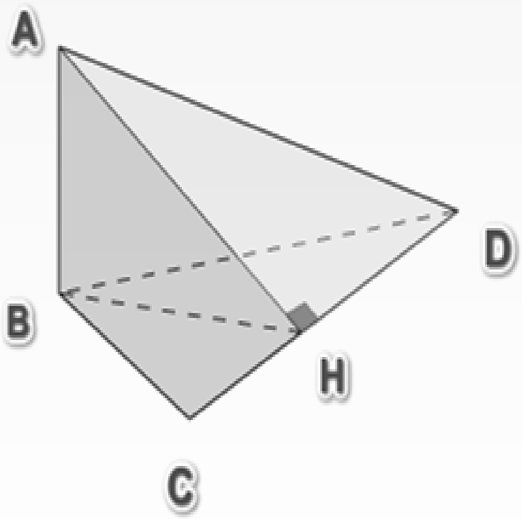
Prove that $\overline{CD} \perp \overline{AH}$.

Solution

Since $\overline{AB} \perp \overline{CD}$ & $\overline{AH} \perp \overline{CD}$,

Therefore $\overline{CD} \perp$ the plane ABH,

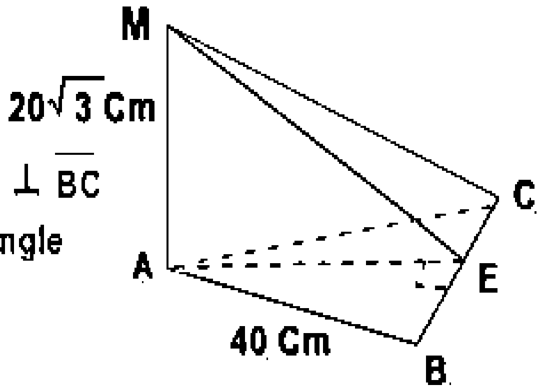
as a result $\overline{CD} \perp \overline{AH}$.



MABC is a triangular pyramid in which Δ ABC is equilateral with side length 40cm. AM $\perp$ plane ABC. AM = $20\sqrt{3}$ cm. E midpoint of BC .

1-Prove that the plane MAE $\perp$ ABC.

2-Calculate m($\angle$ M- $\overleftrightarrow{BC}$-A)



Since $\overleftrightarrow{MA} \perp$ plane ABC & $\overleftrightarrow{MA} \subset$ plane MAB
Therefore plane. MAE $\perp$ plane. ABC.
Since $\overleftrightarrow{AE}$ is the projection of $\overline{ME}$ on plane. ABC & $\overline{AE} \perp \overline{BC}$
Therefore $\overline{ME} \perp \overline{BC}$ Therefore $\angle$ MEA is the plane. angle of $\angle M - \overleftrightarrow{BC} - A$,
Since AE = 40 sin 60° Therefore AE = $20\sqrt{3}$ cm

so , tan ($\angle$MEA) = $\dfrac{20\sqrt{3}}{20\sqrt{3}}$ = 1

i.e. m $\angle$MEA = 45°
Or m $\angle M - \overleftrightarrow{BC} - A = 45°$