Choose the correct answer

(1) At one of the university faculties if a student studies 8 subjects and he can not Pass to the following year unless he succeeds in at least 6 subjects then how many possible ways he may have to pass to the following year ?

ⓐ 56 ⓑ 4 ⓒ 37 ⓓ 14

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(2) If we want to form a committee formed from 4 person chosen from 9 men and 3 women such that the committee includes one woman at least then the number of ways is .....

ⓐ 495 ⓑ 11880 ⓒ 369 ⓓ 252

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(3) Number of diagonals in a polygon has twelve sides is ....

ⓐ 120 ⓑ 132 ⓒ 66 ⓓ 54

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(4) If the points A,B,C ∈ the straight line L_1 and M,N,H ,Z∈ the straight line L_2 then number of triangles can be drawn using the set of points {A,B,C,M,N,H,Z} equals....

ⓐ 210 ⓑ 60 ⓒ 35 ⓓ 30

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(5) If the number of triangles can be drawn using the vertices of a polygon equals 56 triangles Then the number of vertices of this polygon is......

ⓐ 6 ⓑ 7 ⓒ 8 ⓓ 9

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(6) Number of 4 different digits numbers can be formed using the element of the set {0,1,2,3,4} is ......

ⓐ 120 ⓑ 96 ⓒ 5 ⓓ 16

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(7) If $N \in Z^+$ such that ${}^{n}C_{٣} + ٢\,{}^{n}C_{٢} + {}^{n}C_{١} = ١٢٠$ then n=...

ⓐ 7 ⓑ 8 ⓒ 9 ⓓ 10

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(8) If $N \in Z^+$ then ${}^{n}P_{٢}$ my be equal to ....

ⓐ 24 ⓑ 25 ⓒ 27 ⓓ 30

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(9) If ${}^{17}C_{r^2+1} = {}^{17}C_{3r^2}$ then r=......

ⓐ 2 ⓑ −2 ⓒ ±2 ⓓ 4

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(10)If ${}^{x+y}P_2 = 210$, ${}^{y-3}C_3 = 35$ then $\lfloor 2x - y \rfloor$ = ...

ⓐ 5 ⓑ 10 ⓒ 2 ⓓ 1

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(11)If ${}^{n}C_{3} : {}^{n-1}C_{4} = 8:5$ then the value of n equals ....

ⓐ 5 ⓑ 7 ⓒ 8 ⓓ 9

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(12)If the two middle terms in the expansion $(a+2b)^{2n+1}$ are equal then ......

ⓐ $\frac{a}{b}=\frac{1}{2}$ ⓑ $a=4b$ ⓒ $a=8b$ ⓓ $a=2b$

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(13)in the expansion $x^{3}(1+x)^{7}$ the coefficient of the term contain x^{4} is ......

ⓐ ${}^{7}C_{4}$ ⓑ ${}^{7}C_{3}$ ⓒ ${}^{7}C_{1}$ ⓓ 21

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(14)If the term free of X in the expansion $\left(X+\frac{1}{X}\right)^{n}$ is T_7 then n=.....

ⓐ 6 ⓑ 10 ⓒ 12 ⓓ 8

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(15)In the expansion of $(1-X)^{12}$ coefficient of the sixth term : coefficient of the fifth term equals ....

ⓐ 8 : 5 ⓑ 5 : 8 ⓒ −8 : 5 ⓓ −5 : 8

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(16)If $(1+X)^{n} = 1 + a_1X + a_2X^2 + a_3X^3 + \cdots, \frac{a_2+a_3}{a_2} = 3$ then n =....

ⓐ 4 ⓑ 6 ⓒ 8 ⓓ 9

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(17)The sum of coefficients o the expansion $(1+X-3X^2)^{2018}$ *equals* .....

ⓐ −1 ⓑ 1 ⓒ zero ⓓ 2017

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(18)In the expansion of $\left(\sqrt[3]{3}+\sqrt{2}\right)^{5}$ the term which does not contains irrational number

ⓐ 30 ⓑ 40 ⓒ 50 ⓓ 10

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(19)In the expansion of $\left(2+\frac{X}{3}\right)^{n}$ if the coefficient of X^7 , X^8 are equal then n=......

ⓐ 56 ⓑ 55 ⓒ 45 ⓓ 15

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(20)In the expansion of $(X+Y)^{n}$ if the seventh term is the term has the greatest coefficient then (n) equals ....

ⓐ 12 ⓑ 13 ⓒ 14 ⓓ 15

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(21)In the expansion of $X^4\left(X-\frac{1}{X}\right)^9$ according to the descending power of X then the forth term from end equals ....

ⓐ 84x ⓑ −84x ⓒ $84x^7$ ⓓ $-84x^7$

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(22)If $Z=\left(1+\sqrt{3}i\right)^n$, $|Z|=8$ then the principle amplitude of the number z is ....

ⓐ $\frac{\pi}{2}$ ⓑ $\frac{\pi}{3}$ ⓒ $\frac{\pi}{6}$ ⓓ π

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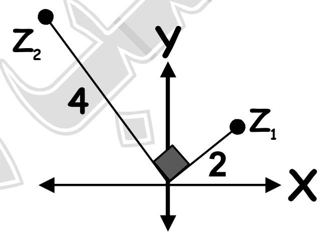
(23)In the opposite figure If Z_1 , Z_2 are two complex numbers then $\left(\frac{Z_2}{Z_1}\right)^2=\cdots$



ⓐ 4 ⓑ −4

ⓒ $4i$ ⓓ $-4i$

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(24) If $Z=-1-i$ then the exponential form of the number Z is......

ⓐ $\sqrt{2}e^{\frac{3\pi}{4}i}$ ⓑ $\sqrt{2}e^{\frac{5\pi}{4}i}$ ⓒ $\sqrt{2}e^{\frac{-3\pi}{4}i}$ ⓓ $\sqrt{2}e^{225i}$

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(25)$(a+b\omega+a\omega^2)(a+b\omega^2+a\omega^4)=$........

ⓐ 1 ⓑ a−b ⓒ $(a-b)^2$ ⓓ b^2-a^2

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(26) $\frac{a-b\omega}{a\omega^2-b}-\omega^2=\cdots$

ⓐ $3i$ ⓑ $\pm\sqrt{3}i$ ⓒ −3 ⓓ 3

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(27)If $(1+\omega)^7=a+b\omega$ where a, b are two real numbers then (a , b) =.....

ⓐ (0 , -1) ⓑ (1 , 1) ⓒ (0 , 1) ⓓ (1 , -1)

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(28)$\sum_{r=0}^{6}\left(1+\omega^r\right)=$.....

ⓐ 7 ⓑ 6 ⓒ 1 ⓓ $1+\omega$

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(29)The conjugate of $1+\omega$ is ....

ⓐ $1-\omega$ ⓑ $1+\omega^2$ ⓒ $1-\omega^2$ ⓓ $-1-\omega$

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(30)The summation of the roots of the equation $(Z-2)^3=1$ equals

ⓐ zero ⓑ 2 ⓒ 1 ⓓ 6

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(31)If $|Z| = |Z - 2|$ then the real part of the number Z equals.

ⓐ 1 ⓑ −1 ⓒ 2 ⓓ −2

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(32) $e^{\theta i} + e^{-\theta i} = ...$

ⓐ $e^{2\theta i}$ ⓑ $2\cos\theta$ ⓒ $2\sin\theta$ ⓓ $e^{-2\theta i}$

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(33) $i^{12} + i^{13} + i^{14} + \ldots + i^{112} = \ldots$

ⓐ i ⓑ -1 ⓒ 1 ⓓ $-i$

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(34) If $|Z| = 10$ then $Z\bar{Z} = ...$

ⓐ 10 ⓑ 1 ⓒ 100 ⓓ -100

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(35)If $Z = X + iY$ then the real part of the number e^Z is ......

ⓐ $e^x \cos y$ ⓑ $e^x \sin y$ ⓒ e^x ⓓ $\cos y$

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(36) The amplitude of the number $(1 - \cos\theta) + i \sin\theta$ Where $0 < \theta < \pi$ is ....

ⓐ $\frac{\pi}{4} - \frac{\theta}{2}$ ⓑ $\frac{\pi}{2} - \frac{\theta}{2}$ ⓒ $\frac{\pi}{2} - \theta$ ⓓ $\frac{\pi}{2} - \frac{\theta}{4}$

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(37) $\begin{vmatrix} \omega & i \\ i & \omega \end{vmatrix} = ...$

ⓐ 1 ⓑ −1 ⓒ ω ⓓ $-\omega$

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(38) If each Of A and B is singular matrix then $(AB)^{-1} = ...$

ⓐ $-AB$ ⓑ $A^{-1}B^{-1}$ ⓒ $B^{-1}A^{-1}$ ⓓ $(BA)^{-1}$

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(39) If $A = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 8 & 16 \end{pmatrix}$ then $R(A) = ...$

ⓐ zero ⓑ 1 ⓒ 2 ⓓ 3

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(40) If $A = \begin{pmatrix} 1 & -2 & 3 \\ K & 0 & 1 \\ 3 & 2 & -1 \end{pmatrix}$ and $R(A) = 2$ then k=.....

ⓐ zero ⓑ 2 ⓒ −2 ⓓ 6

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(41) Number of solutions for the system $2x+5y=0\ ,\ 3x-z=0\ ,\ 2y-3z=0$ is .....

ⓐ zero solution only ⓑ infinite number of solution including zero

ⓒ Zero ⓓ infinite number of solution not including

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(42) The ystem $\begin{pmatrix} 2 & 2 & 3 \\ 1 & -2 & -3 \\ 4 & -2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \square$

ⓐ zero solution only ⓑ infinite number of solution including zero

ⓒ Zero ⓓ infinite number of solution not including

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(43) $\begin{vmatrix} a+b & c+b & a+c \\ c & a & b \\ 1 & 1 & 1 \end{vmatrix} = \ldots$

ⓐ −1 ⓑ zero ⓒ $a+b+c$ ⓓ abc

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(44) the equations $x+2y+3z=5\ ,\ 2x-3y+kz=13\ ,\ 3x+ky+2z=3$ has one solution then $k\in$ .....

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{-1\}$ ⓒ $\mathbb{R}-\{13\}$ ⓓ $\mathbb{R}-\{-1\ ,\ 13\}$

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(45) if $A=\begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 2 & -2 & 1 \end{pmatrix}$ then $R\left(A^T\right)=$ .....

ⓐ zero ⓑ 1 ⓒ 2 ⓓ 3

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(46) If the matrix A is of order $m\times n$ *then* ...

ⓐ $R(A)\le$ the smallest number of m , n

ⓑ $R(A)\prec$ the smallest number of m , n

ⓒ $R(A)\ge$ the smallest number of m , n

ⓓ $R(A)\succ$ the smallest number of m , n

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(47) The sum of roots of the equation $\begin{vmatrix} x & 0 & 0 \\ 1 & x & 0 \\ 2 & 3 & x \end{vmatrix} = 8$ in $\mathbb{C}$ is ……

ⓐ zero ⓑ2 ⓒ 4 ⓓ 8

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(48) If the two equations $2x + y = 1$, $4x + 2y = k$ have infinite numbers of solution then $k = \ldots\ldots$

ⓐ zero ⓑ 1 ⓒ 2 ⓓ 3

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(49) If X is a complex number then number of solution of the equation $\begin{vmatrix} x^3+1 & x-1 \\ x+1 & x^3-1 \end{vmatrix} = 0$ equals

ⓐ 6 ⓑ 5 ⓒ 4 ⓓ 3

Producing answers questions

Q(1) In the expansion of $\left(4X^2+\frac{1}{2X}\right)^{15}$ according to the descending power of X find the value of the term free of X and if the two middle terms are equals find(X) value

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Q(2) In the expansion of $(X+Y)^n$ if T_1, T_2, T_3 form an arithmetic sequence , and T_2 arithmetic mean between T_1 , T_3 and if X=2Y find the value of n

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Q(3) In the expansion of $(1+Y)^n$ If $3T_4, \sqrt{5}\,T_6, 6T_8$ are in geometric sequence find the value of n .

Q(4) Find the coefficient of $\frac{1}{X^5}$ *in the expansion of* $\frac{1}{X^3}\left(X+\frac{1}{X^2}\right)^{10}$ then prove that this expansion does not contain a term free of X .

Q(5)If T_2, T_3, T_4 in the expansion of $(X+a)^n$ are 18,144,672 respectively find the value

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Q(6) If $(a-X)^{14}=C_0+C_1X+C_2X^2+C_3X^3+\cdots.+C_{14}X^{14}$ And If $4C_4+11(C_3+C_2)=0$ find the value of a .

Q(7)If the ratio between the fifth term in the expansion of $\left(X+\frac{1}{X}\right)^{15}$ and the fourth term in the expansion of $\left(X-\frac{1}{X^2}\right)^{14}$ equals -16:15 find the value of X

Q(8) Find the greatest term in the expansion of $(2X+3Y)^{10}$

Q(9) In the expansion of $\left(X^2+\frac{1}{X}\right)^{3n}$ prove that :the term free of X equals the coefficient of the term contains X^{3n} and if n=6 find the ratio between the term free of X and the coefficient off the middle term

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Q(10) In the expansion of $\left(aX^2+\frac{1}{aX}\right)^{11}$ if the coefficients of X^7 , the coefficient of X^4 are equal find the value of (a)

Q(11) In the expansion of $\left(X^m+\frac{1}{X}\right)^6$ such that: $m \in Z^+$ find the value of m find the value of which makes the expansion has a term free of X

Q(12) In the expansion of $\left(\sqrt{X}+\frac{1}{X}\right)^8$ If $T_4, T_5, 25T_7$ are proportional then find the value of X

Q(13) If n is a positive integer prove that: there is no term free of X in the expansion of $\left(X^5+\frac{1}{X^2}\right)^n$ unless if n is multiple of the number 7 then find this term when n=7

Q(14) If $\sqrt{3}(Z-1)=i(Z+1)$ put the number Z in the exponential form then find its cubic roots in the exponential form and Represent on Argend's diagram

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Q(15) If $Z=\frac{1+\sqrt{3}i}{1+i}$ put the number Z in the trigonometric form then De Moivre's theorem to Prove that : $Z^6=8i$

Q(16) If $(1-i)X+(1+i)Y=2i$ where X , Y ∈ R find the value of X , Y find the different value of the number $z^{\frac{8}{3}}$ $where\ Z=X+yi$ in the trigonometric form

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Q(17) If $Z_1=4\left(sin\frac{5\pi}{6}+isin\frac{5\pi}{6}\right)$, $Z_2=i$, $Z_3=2e^{\frac{5\pi}{6}}$ and $Z=\frac{Z_1}{Z_2\times Z_3}$ find Z in the trigonometric form then find its square roots

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Q(18) put the number $Z=\sqrt{2}\left(\frac{1+itan\frac{\pi}{12}}{1-itan\frac{\pi}{12}}\right)$ in the trigonometric form then find the value of $Z^{\frac{4}{3}}$ in the trigonometric form

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Q(19) if the number $Z_1=1-\sqrt{3}i$ and $\frac{Z_2}{Z_1}=4\times e^{\pi i}$ find the two square roots of the number Z_2

Q(20) If $Z_1=\frac{6+4i}{1+i}$, $Z_2=\frac{26}{5+i}$ prove that the two numbers Z_1 , Z_2 are conjugate then find the cubic roots of the number $Z=4(Z_1-Z_2)$

Q(21) If the number $Z=\frac{1+\omega}{(1-i)^2}+\frac{1-\omega}{(1+i)^2}$ find the modules and the principle amplitude of the number Z where $\omega=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Q(22) Solve the equation $Z^2-2\overline{Z}=0$ in set of complex numbers

Q(23) Use De Moivre's theorem to find the roots of the equation $Z^3 + 27i = 0$

Q(24)If the amplitude of $(Z + i) = \frac{\pi}{4}$ and the amplitude of $(Z - 3) = \frac{3\pi}{4}$ find the number Z in the trigonometric form

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Q(25) If $Z_1 = cos75^o + isin75^o$ and $Z_2 = cos15^o + isin15^o$
Find in the trigonometric form the number $Z_1 + Z_2$

Q(26) $Z_1 = cos114^o + isin66^o$, $Z_2 = cos42^o + isin138^o$
, $Z_3 = cos24^o + isin114^o$ Find the number Z= $\frac{Z_1 Z_2}{Z_3}$ in the algebraic form

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Q(27) Find the forth roots of the number (-1) and represent it in argend's plane

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Q(28) If $\frac{7-11i}{4+i} = a + bi$ find all possible value of the expression $\left(\sqrt{-b} + ai\right)^{\frac{3}{2}}$

Q(29)If $1, \omega, \omega^2$ are the cubic roots of unity prove that :

① $\frac{5+2\omega}{2+3\omega} + \frac{5+2\omega^2}{2+3\omega^2} = \frac{13}{7}$ ① $\left(5 - \frac{5}{1+\omega^2} + \frac{3}{\omega^2}\right)^6 = 64$

Q(30)If $1, \omega, \omega^2$ are the cubic roots of unity prove that :

① $\frac{X+Y\omega}{X\omega^2+Y} + \frac{Z\omega^2+L}{Z+L\omega} = -1$ ① $\left(\frac{\omega}{1+2\omega}\right)^2 + \left(\frac{\omega^2}{1+2\omega}\right)^2 = \frac{1}{3}$

Q(31)without expanding the determinant prove that :

$$\begin{vmatrix} 1 & a & a \\ 1 & b & a \\ 1 & c & -b \end{vmatrix} = (a - b)(a + b)$$

Q(32)without expanding the determinant prove that :

$$\begin{vmatrix} (a+b)^2 & ab & a^2 + b^2 \\ (c+d)^2 & cd & c^2 + d^2 \\ (n+h)^2 & nh & n^2 + h^2 \end{vmatrix} = zero$$

Q(33)without expanding the determinant prove that : $\begin{vmatrix} 1 & i & \omega \\ \omega i & -\omega & 0 \\ \omega^2 i & -\omega^2 & \omega \end{vmatrix} = zero$

Q(34) If X is one factor of the factor of $\begin{vmatrix} 4 & 3 & 2K \\ 1 & k & X \\ X+3 & X+2 & 2K \end{vmatrix}$ find the value of K

Q(35) If $\begin{vmatrix} X+2 & Y & Z+2 \\ X & Y+2 & Z \\ X & Y & Z+2 \end{vmatrix} = -4$ find the value of X+Y+Z

Q(36) without expanding the determinant prove that :

$\begin{vmatrix} 3X & 3X & 3X \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix} = zero$

Q(37)use the determinant properties to find the S.S of the equation

$\begin{vmatrix} 1 & -X & 0 \\ X & 1 & X \\ 1 & -1 & X+1 \end{vmatrix} = \begin{vmatrix} X^2 & 1 \\ -X & X \end{vmatrix}$

Q(38) Prove that : $\begin{vmatrix} X & a & a \\ a & X & a \\ a & a & X \end{vmatrix} = (X+2a)(X-a)^2$

Q(39)use the determinant properties to find the S.S of the equation

$\begin{vmatrix} a & b & c \\ b & a+b & a+b+c \\ b & a & c \end{vmatrix} + \begin{vmatrix} a & b & 0 \\ b & a+b & b \\ b & a+b+c & a+b \end{vmatrix} = zeo$

Q(40)without expanding the determinant prove that : $\begin{vmatrix} 1 & 1 & 1 \\ 1+Y & 1 & 1 \\ 1 & 1+Y & 1 \end{vmatrix} = Y^2$

Q(41) Solve the following equations : 2X+2Y-Z=3 , 3X+Y=5 , X+Y+2Z=9
Using the multiplicative inverse for matrices

Q(42) If $A = \begin{pmatrix} 3 & 1 & 1 \\ K & 4 & 10 \\ 1 & 7 & 17 \end{pmatrix}$ find the value of K which makes Rank (A) minimum as possible

Q(43) show that the system
$2X + 3Y + 5Z = 0 , 7X + 4Y - 2Z = 0 , 6X + 9y + 15z = 0$
Has infinite number of solution then write the general form of solution

Multiple choice answers

(1)	c	(2)	c	(3)	d	(4)	d	(5)	c
(6)	b	(7)	b	(8)	d	(9)	c	(10)	d
(11)	c	(12)	d	(13)	c	(14)	c	(15)	c
(16)	c	(17)	b	(18)	d	(19)	b	(20)	a
(21)	a	(22)	d	(23)	b	(24)	c	(25)	c
(26)	b	(27)	d	(28)	b	(29)	b	(30)	d
(31)	a	(32)	b	(33)	c	(34)	c	(35)	a
(36)	b	(37)	d	(38)	c	(39)	b	(40)	b
(41)	a	(42)	b	(43)	b	(44)	d	(45)	d
(46)	a	(47)	a	(48)	c	(49)	b		

Answers

Q(1) $T_{r+1} = {}^{15}C_r \left(\frac{1}{2}X^{-1}\right)^r (4X^2)^{15-r} \Rightarrow T_{r+1} = {}^{15}C_r\,(2)^{30-3r}(X)^{30-2r}$

The term free of X : $30-2r=0 \quad \Rightarrow r=10 \quad \therefore \textit{The term free of } X \textit{ is } T_{11}$

$\therefore \textit{The term free of } X \textit{ is } T_{11} = {}^{15}C_{10}(2)^{30-30} = 3003$

The two middle terms are $T_8 = T_9 \quad \therefore \frac{T_9}{T_8} = 1 \quad \therefore \frac{15-8+1}{8} \times \frac{4X^2}{\frac{1}{2X}} = 1 \quad \therefore X = \frac{1}{2}$

Q(2) $\because T_1, T_2, T_3$ in (A.S) $\Rightarrow 2T_1 = T_2 + T_3 \quad \div\ T_2$

$\therefore \frac{T_1}{T_2} + \frac{T_3}{T_2} = 2 \quad \Rightarrow \frac{1}{n-1+1} \times \frac{X}{Y} + \frac{n-2+1}{2} \times \frac{Y}{X} = 2 \quad \because X = 2Y$

$\therefore \frac{1}{n} \times \frac{X}{Y} + \frac{n-1}{2} \times \frac{Y}{X} = 2 \quad \because X = 2Y \Rightarrow \quad \frac{1}{n} \times \frac{2Y}{Y} + \frac{n-1}{2} \times \frac{Y}{2Y} = 2$

$\therefore \frac{2}{n} + \frac{n-1}{4} = 2 \quad \times 4n \quad \therefore 8 + n^2 - n = 8n \quad \Rightarrow n^2 - 9n + 8 = 0$

$\therefore (n-1)(n-8) = 0 \quad \therefore n = 8 \quad or \quad \boxed{n=1}$ refused

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Q(3)

$\frac{6}{\sqrt{5}} \times \frac{T_8}{\sqrt{5}T_6} = \frac{\sqrt{5}T_6}{3T_4} \quad \therefore \quad \frac{6}{\sqrt{5}} \times \frac{T_8}{T_7} \times \frac{T_7}{T_6} = \frac{6}{\sqrt{5}} \times \frac{T_6}{T_5} \times \frac{T_5}{T_4}$

$\frac{n-7+1}{7} \times \frac{X}{1} \times \frac{n-6+1}{6} \times \frac{X}{1} = \frac{5}{18} \times \frac{n-5+1}{5} \times \frac{X}{1} \times \frac{n-4+1}{4} \times \frac{X}{1}$

$\therefore 12(n-6)(n-5) = 7(n-4)(n-3) \Rightarrow 5n^2 - 83n + 286 = 0 \Rightarrow n = 12$

Q(4) In expansion $\left(X + \frac{1}{X^2}\right)^{10} \Rightarrow T_{r+1} = {}^{15}C_r\,(X)^{10-3r}$

In expansion $\frac{1}{X^3}\left(X + \frac{1}{X^2}\right)^{10} \Rightarrow T_{r+1} = X^{-3}\,{}^{15}C_r\,(X)^{10-3r} = {}^{15}C_r\,(X)^{7-3r}$

Coefficient of $\frac{1}{X^5}$ $\therefore 7 - 3r = -5 \quad \therefore r = 4 \quad \therefore$ Coefficient $= {}^{15}C_4 = 210$

The term free of X : $7 - 3r = 0 \quad \therefore r = \frac{7}{3} \notin Z^+$ $\therefore$ there is no term free of X

Q(5) $\because T_4 : T_3 = 672 : 144 \quad \therefore \frac{n-2}{3} \times \frac{a}{X} = \frac{14}{3} \quad \times 3X \quad \therefore (n-2)a = 14X \rightarrow (1)$

$\because T_3 : T_2 = 144 : 18 \quad \therefore \frac{n-1}{2} \times \frac{a}{X} = 8 \quad \times 2X \quad \therefore (n-1)a = 16X \rightarrow (2)$

By dividing 1,2 $\therefore \frac{n-2}{n-1} = \frac{7}{8} \quad \therefore 8n - 16 = 7n - 7 \quad \therefore n = 9$

$\because T_2 = 18 \quad \therefore {}^9C_1 \times a \times X^8 = 18 \quad \therefore aX^8 = 2 \rightarrow (3)$

From (2) $a = 2X \quad \therefore 2X^9 = 2 \quad \therefore X = 1\ ,\ a = 2$

Q(6)

$4C_4 + 11(C_3 + C_2) = 0 \quad \div C_3$

$$4 \times \frac{\text{Coefficient of } T_5}{\text{Coefficient of } T_4} + 11 \times \left(1 + \frac{\text{Coefficient of } T_3}{\text{Coefficient of } T_4}\right) = 0$$

$$4 \times \frac{14 - 4 + 1}{4} \times \frac{-1}{a} + 11 \times \left(1 + \frac{3}{14 - 3 + 1} \times \frac{a}{-1}\right) = 0$$

$$\therefore \frac{-11}{a} + 11 \times \left(1 - \frac{a}{4}\right) = 0 \quad \times -\frac{4a}{11}$$

$\therefore 4 - 4a + a^2 = 0 \quad \therefore (a-2)^2 = 0 \quad \therefore a = 2$

Q(7) $\frac{T_5}{T_4} = \frac{-16}{15} \Rightarrow \frac{{}^{15}C_4\,(X^{-1})^4 X^{11}}{{}^{15}C_3\,(-X^{-2})^3 X^{11}} = \frac{-16}{15} \quad \therefore \frac{-15}{4} X^2 = \frac{-16}{15}$

$\therefore X^2 = \frac{64}{225} \quad \therefore X = \pm \frac{8}{15}$

Q(8)

Put $\frac{T_{r+1}}{T_r} \geq 1 \quad \Rightarrow \frac{10-r+1}{r} \times \frac{3}{2} \geq 1$

$\therefore \frac{11-r}{r} \geq \frac{2}{3} \quad \Rightarrow 33 - 3r \geq 2r \quad \therefore 5r \leq 33 \quad \therefore r \leq 6.6 \quad \therefore r = 6$

Coefficient of T_7 is the greatest Coefficient $= {}^{10}C_6 \times 3^6 \times 2^4 = 2449440$

Q(9)

$T_{r+1} = {}^{3n}C_r\ (X^{-1})^r (X^2)^{3n-r} \Rightarrow T_{r+1} = {}^{3n}C_r\ X^{6n-3n}$

The term free of X

$6n - 3r = 0\ \therefore r = 2n$

$\therefore$the term free of X is T_{2n+1}

$\therefore T_{2n+1} = {}^{3n}C_{2n} = {}^{3n}C_n$

Coefficien of X^{3n}

$6n - 3r = 3n \Rightarrow r = n$

Coefficien of $X^{3n} = {}^{3n}C_n$

When n=6 $\Rightarrow$ $middle\ term\ is T_{10}$, term free of X is T_{13}

Q(10)

$T_{r+1} = {}^{11}C_r\ \left(\frac{1}{a}X^{-1}\right)^r (aX^2)^{11-r} \Rightarrow T_{r+1} = {}^{11}C_r\ (a)^{11-2r} X^{22-3r}$

Coefficien of X^7

$22 - 3r = 7 \quad \Rightarrow r = 5$

$\therefore$ Coefficien of $X^7 = {}^{11}C_5\ a$

Coefficien of X^4

$22 - 3r = 4 \quad \Rightarrow \quad r = 6$

$\therefore$ Coefficien of $X^4 = {}^{11}C_6\ a^{-1}$

$\because$coefficent of X^7 equals coefficient of X^4

$\therefore {}^{11}C_5\ a = {}^{11}C_6\ a^{-1} \quad \therefore a = \frac{1}{a} \quad \therefore a = \pm 1$

Q(11) $T_{r+1} = {}^6C_r\ (X^{-1})^r (X^m)^{6-r} \Rightarrow T_{r+1} = {}^6C_r\ X^{6m-mr-r}$

$\therefore 6m - mr - r = 0 \quad \Rightarrow 6m = mr + r \quad \Rightarrow r = \frac{6m}{m+1}\ , \quad \because r \in Z^+$

$\therefore (m+1)$ must be divisible by 6

$m + 1 = 1 \quad or \quad m + 1 = 2 \quad or \quad m + 1 = 2 \quad or \quad m + 1 = 6$

$\boxed{m = 0\ ✗}$ $\boxed{m = 1\ ✓}$ $\boxed{m = 2\ ✓}$ $\boxed{m = 5\ ✓}$

Q(12) $\frac{T_6}{25T_7} = \frac{T_5}{T_4} \Rightarrow \frac{1}{25} \times \frac{T_6}{T_7} = \frac{T_5}{T_4}$

$\therefore \frac{1}{25} \times \frac{6}{8-6+1} \times \frac{\sqrt{X}}{\frac{1}{X}} = \frac{8-4+1}{4} \times \frac{\frac{1}{X}}{\sqrt{X}} \Rightarrow X\sqrt{X} = \frac{5}{4} \times \frac{1}{X\sqrt{X}}$

$\therefore \left(X\sqrt{X}\right)^2 = \frac{125}{8} \quad \Rightarrow X^3 = \frac{125}{8} \qquad \therefore \boxed{X = \frac{5}{2}}$

Q(13) $T_{r+1} = {}^nC_r\ (X^{-2})^r \left(X^5\right)^{n-r} \Rightarrow T_{r+1} = {}^nC_r\ X^{5n-7r}$

$\therefore 5n - 7r = 0 \Rightarrow r = \frac{5n}{7} \in Z^+$

$\frac{5n}{r}$ must be multiple of 7 $\quad when\ n = 7\ \therefore r = 5$

The term free of X is $T_6 = {}^7C_5 = 21$

Q(14) $\because \sqrt{3}(Z-1)=i(Z+1) \Rightarrow \sqrt{3}Z-\sqrt{3}=Zi+i$

$\therefore \sqrt{3}Z-iZ=\sqrt{3}+i \quad \Rightarrow Z(\sqrt{3}-i)=\sqrt{3}+i \quad \therefore Z=\frac{\sqrt{3}+i}{\sqrt{3}-i}$

$\therefore Z=\frac{\sqrt{3}+i}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i}=\frac{1}{2}+\frac{\sqrt{3}}{2}i \quad \therefore |Z|=1 \ ,\theta=60^o=\frac{\pi}{3}$

$\therefore Z=e^{\frac{\pi}{3}i} \quad \therefore \sqrt[3]{Z}=e^{\frac{\frac{\pi}{3}+2K\pi}{3}i}$

$$\therefore \begin{cases} K=0 & \Rightarrow \sqrt[3]{Z}=e^{\frac{\pi}{9}i} \\ K=1 & \Rightarrow \sqrt[3]{Z}=e^{\frac{7\pi}{9}i} \\ K=2 & \Rightarrow \sqrt[3]{Z}=e^{\frac{13\pi}{9}i}=e^{\frac{-5\pi}{9}i} \end{cases}$$

Q(15) $Z=\frac{1+\sqrt{3}i}{1+i} \Rightarrow Z=\frac{2(cos60^o+isin60^o)}{\sqrt{2}(cos45^o+isin45^o)}=\sqrt{2}(cos15^o+isin15^o)$

$\therefore Z^6=(\sqrt{2})^6(cos6\times15^o+isin6\times15^o)=8(cos90^o+isin90^o)$

$\therefore Z^6=8(0+i\times1)=8i$

Q(16) $(1-i)X+(1+i)Y=2i \Rightarrow X-iX+Y+iY=2i$

$\therefore X+Y+(Y-X)i=2i \Rightarrow \begin{cases} X+Y=0 \\ X-Y=2 \end{cases} \Rightarrow \begin{cases} X=-1 \\ Y=1 \end{cases}$

$\therefore Z=-1+i \Rightarrow |Z|=\sqrt{2} \ ,\theta=135^o$

$\therefore Z=\sqrt{2}(cos135^o+isin\,135^o) \Rightarrow Z^8=2^4(cos0^o+isin0^o)$

$Z^{\frac{8}{3}}=\sqrt[3]{16}\left(cos\frac{0+2K\pi}{3}+isin\frac{0+2K\pi}{3}\right)$ put K=0 , K=1 , K=2

Q(17) $Z_1=4\left(sin\frac{5\pi}{6}+isin\frac{5\pi}{6}\right) \Rightarrow Z_1=4(sin150^o+icos150^o)$

$Z_1=4(cos(90^o-150^o)+isin(90^o-150^o))=4(cos(-60^o)+isin(-60^o))$

$Z_2=i \quad \Rightarrow Z_2=cos90^o+isin90^o$

$Z_3=i \quad \Rightarrow 2(cos150^o+isin150^o)$

$Z=\frac{Z_1}{Z_2\times Z_3}=2(cos(300^o-90^o-150^o)+isin(300^o-90^o-150^o))$

$\therefore Z=2(cos60^o+isin60^o) \quad \therefore \sqrt{Z}=\sqrt{2}\left(cos\frac{60^o+2\pi K}{2}+isin\frac{60^o+2\pi K}{2}\right)$

Q(18) $Z=\sqrt{2}\left(\frac{1+itan\frac{\pi}{12}}{1-itan\frac{\pi}{12}}\right)$ $\times$ both denominator and numerator by $cos\frac{\pi}{12}$

$\therefore Z=\sqrt{2}\left(\frac{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}}\right)=\sqrt{2}\left(\frac{\cos15^o+i\sin15^o}{\cos15^o-i\sin15^o}\right)$

$Z=\sqrt{2}\left(\frac{\cos15^o+i\sin15^o}{\cos-15^o+i\sin-15^o}\right)$ $\therefore Z=\sqrt{2}(\cos30^o+i\sin30^o)$

$Z^4=4(\cos120^o+i\sin120^o)$

$\therefore Z^{\frac{4}{3}}=4\left(\cos\frac{120^0+2\pi K}{3}+i\sin\frac{120^0+2\pi K}{3}\right)$ then put K=0,1,2

Q(19) $|Z_1|=2$, $\theta=-\tan^{-1}\sqrt{3}=-\frac{\pi}{3}$ $\Rightarrow Z_1=2e^{-\frac{\pi}{3}i}$

$\because \frac{Z_2}{Z_1}=4e^{\pi i}$ $\therefore Z_2=4e^{\pi i}\times 2e^{-\frac{\pi}{3}i}=8e^{\frac{2\pi}{3}i}$

$\therefore \sqrt{Z_2}=2\sqrt{2}e^{\frac{\frac{22\pi}{3}+2K\pi}{2}i}$ $\therefore \sqrt{Z_2}=$

Q(20) $Z_1=\frac{6+4i}{1+i}\times\frac{1-i}{1-i}$ $\therefore Z_1=5-i$

$Z_1=\frac{26}{5+i}\times\frac{5-i}{5-i}$ $\therefore Z_2=5+i$ $\therefore$ the two numbers are conjugate

$Z=4(Z_1-Z_2)=4(5-i-5-i)=-8i$ $\therefore Z$

$=8(\cos(-90^o)+i\sin(-90^o))$

$\sqrt[3]{z}=2\left(\cos\frac{-90+2k\pi}{3}+i\sin\frac{-90+2k\pi}{3}\right)$ Put K =0,1,2

Q(21)

$Z=\frac{1+\omega}{(1-i)^2}+\frac{1-\omega}{(1+i)^2}$ $\Rightarrow$ $Z=\frac{1+\omega}{-2i}+\frac{1-\omega}{2i}$

$\therefore Z=\frac{1-\omega+1-\omega}{2i}$ $\Rightarrow$ $\therefore Z=\frac{-2\omega}{2i}$

$\therefore Z=\frac{-\omega}{i}\times\frac{-i}{i}$ $\Rightarrow$ $\therefore Z=\omega i$

$\therefore Z=i\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ $\Rightarrow$ $\therefore Z=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$\therefore |Z|=1$, $amplitude=-150^o$ $\Rightarrow$ $\therefore Z=\cos(-150^o)+i(\sin)(-150^o)$

Q(22) $Let\ z=X+Yi\ then\ \overline{Z}=X-yi$

$Z^2-2\overline{Z}=0$ $\Rightarrow$ $(X+iY)^2-2(X-iY)=0$

$\therefore X^2 - Y^2 + 2YXi - 2Yi + 2iY = 0 \quad \Rightarrow \quad \therefore \underline{X^2 - Y^2 - 2X} + \underline{2Y(X+1)}i = 0$

$\therefore X^2 - Y^2 - 2X = 0 \rightarrow (1) \qquad \therefore 2Y(X+1) = 0 \rightarrow (2)$

$\therefore Y = 0 \quad or \ Y = 1 \ \therefore X^2 - 2X = 0 \quad \Rightarrow$

Q(23) $Z^3 + 27i = 0 \quad \therefore Z^3 = -27i \quad \therefore Z^3 = 27(\cos(-90^o) + i\sin(-90^o))$

$\therefore Z = \sqrt[3]{27}\left(\cos\frac{-90^o + 2K\pi}{3} + i\sin\frac{-90^o + 2K\pi}{3}\right)$ put K=0,1,2

Q(24)let $Z = X + iY \quad \therefore X + i = X + iY + i \quad \therefore Z + i = X + (Y+1)i$

$\because amplitude\ of(Z+i) = \frac{\pi}{4} \quad \therefore \frac{Y+1}{X} = \tan 45^o \quad \therefore \boxed{X = Y + 1 \rightarrow (1)}$

$Z - 3 = X + iY - 3 \quad \therefore Z - 3 = (X-3) + iY$

$\because amplitude\ of(Z-3) = \frac{3\pi}{4} \quad \therefore \frac{Y}{Y-3} = \tan 135^o \quad \therefore \boxed{Y = 3 - X \rightarrow (2)}$

Solving 1,2 $\boxed{\therefore X = 2\ ,\ Y = 1 \quad \therefore Z = 2 + i}$

Q(25)$\because Z_1 + Z_2 = \cos 75^o + i\sin 75^o + \cos 15^o + i\sin 15^o$

$Z_1 + Z_2 = \cos 75^o + \cos 15^o + i\sin 75^o + +i\sin 15^o$

$Z_1 + Z_2 = (\cos 75^o + \sin 75^o) + i(\sin 75^o + +i\cos 75^o)$

$|Z_1 + Z_2| = \sqrt{(\cos 75 + \sin 75)^2 + (\sin 75 + \cos 75)^2}$

$|Z_1 + Z_2| = \sqrt{2(\cos 75 + \sin 75)^2} = \sqrt{2(\cos^2 75^o + \sin^2 75^o) + 2\sin 75^o \cos 75^o}$

$\therefore |Z_1 + Z_2| = \sqrt{2(1 + \sin 150^o)} = \sqrt{3}$

$\tan\theta = \frac{\sin 75^o + \cos 75^o}{\cos 75^o + \sin 75^o} \quad \therefore \theta = 45^o \quad \therefore Z_1 + Z_2 = \sqrt{3}(\cos 45^o + i\sin 45^o)$

Q(26)$Z_1 = \cos 114^o + i\sin(180^o - 114^o) = \cos 114^o + i\sin 114^o$

$Z_2 = \cos 114^o + i\sin(180^o - 114^o) = \cos 114^o + i\sin 114^o$

$Z_3 = \cos(90^o - 66^o) + i\sin(180^o - 66^o) = \cos 66^o + i\sin 66^o$

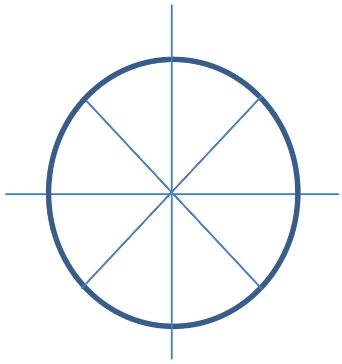
$\therefore Z = \cos(114^o + 42^o - 66^o) + i\sin(114^o + 42^o - 66^o)$

$\therefore Z = \cos 90^o + i\sin 90^o = i$

Q(27) $Z = -1 \quad \therefore Z = \cos 180^o + i\sin 180^o$

$\therefore \sqrt[4]{Z} = \cos\frac{180^o + 2K\pi}{4} + i\sin\frac{180^o + 2K\pi}{4}$

$$\begin{cases} \cos 45^o + \sin 45^o \\ \cos 135^o + \sin 135^o \\ \cos -135^o + \sin -135^o \\ \cos -45^o + \sin -45^o \end{cases}$$



Q(28) $\frac{7-11i}{4+i}=a+bi \quad \Rightarrow \frac{7-11i}{4+i}\times\frac{4-i}{4-i}=a+bi \quad \Rightarrow 1-3i=a+bi$

$\therefore \sqrt{-b}+ai=\sqrt{3}+i\ =2(\cos30^o+i\sin30^o)$

$\therefore \left(\sqrt{-b}+ai\ \right)^3=8(\cos90^o+i\sin90^o)$

$\therefore \left(\sqrt{-b}+ai\ \right)^{\frac{3}{2}}=2\sqrt{2}\left(\cos\frac{90^o+2K\pi}{2}+i\sin\frac{90^o+2K\pi}{2}\right)$

$\begin{cases} 2\sqrt{2}(\cos45^o+i\sin45^o) \\ 2\sqrt{2}(\cos-135^o+i\sin-135^o) \end{cases}$

Q(29)(1) $L.H.S=\frac{5+2\omega}{2+3\omega}+\frac{5+2\omega^2}{2+3\omega^2}=\frac{(5+2\omega)(2+3\omega^2)+(5+2\omega^2)(2+3\omega)}{(2+3\omega)(2+3\omega^2)}$

$\frac{10+15\omega^2+4\omega+6\omega^3+10+15\omega+4\omega^2+6\omega^3}{4+6\omega^2+6\omega+9\omega^3}$

$\frac{20+19\omega^2+19\omega+12\omega^3}{4+6\omega^2+6\omega+9}=\frac{20-19+12}{4-6+9}=\frac{13}{7}$

(2)L.H.S= $\left(5-\frac{5}{1+\omega^2}+\frac{3}{\omega^2}\right)^6=\left(5-\frac{5}{-\omega}+\frac{3}{\omega^2}\right)^6=(5+5\omega^2+3\omega)^6$

$(-5\omega+3\omega)^6=(-2\omega)^6=(-2)^6\times(\omega)^6=64\times1=64$

Q(30) $l.H.S=\frac{X+Y\omega}{X\omega^2+Y}+\frac{Z\omega^2+L}{Z+L\omega}=\frac{X\omega^3+Y\omega}{X\omega^2+Y}+\frac{Z\omega^2+L\omega^3}{Z+L\omega}$

$=\frac{\omega(X\omega^2+Y)}{(X\omega^2+Y)}+\frac{\omega^2(Z+L\omega)}{(Z+L\omega)}=\omega+\omega^2=-1$

(2) $L.H.S=\left(\frac{\omega}{1+2\omega}\right)^2+\left(\frac{\omega^2}{1+2\omega}\right)^2=\frac{\omega^2}{(1+2\omega)^2}+\frac{\omega^4}{(1+2\omega)^2}$

$=\frac{\omega^2+\omega^4}{(1+\omega+\omega)^2}=\frac{\omega+\omega^2}{(\omega-\omega)^2}=\frac{-1}{\left(\pm\sqrt{3}i\right)^2}=\frac{1}{3}$

Q(31) $\begin{vmatrix} 1 & a & a \\ 1 & b & a \\ 1 & c & -b \end{vmatrix}$ $\qquad C_2^{/}=C_2-aC_1\ ,C_3^{/}=C_3-aC_1$

$\begin{vmatrix} 1 & 0 & 0 \\ 1 & b-a & 0 \\ 1 & c-a & -b-a \end{vmatrix}=(b-a)(-a-b)=(a-b)(a+b)$

Q(32) $\begin{vmatrix} (a+b)^2 & ab & a^2+b^2 \\ (C+d)^2 & cd & c^2+d^2 \\ (n+h)^2 & nh & n^2+h^2 \end{vmatrix}=\begin{vmatrix} a^2+b^2+2ab & ab & a^2+b^2 \\ c^2+d^2+2cd & cd & c^2+d^2 \\ n^2+h^2+2nh & nh & n^2+h^2 \end{vmatrix}$ C_1-C_3

$$\begin{vmatrix} 2ab & ab & a^2+b^2 \\ 2cd & cd & c^2+d^2 \\ 2nh & nh & n^2+h^2 \end{vmatrix} = 2\begin{vmatrix} ab & ab & a^2+b^2 \\ cd & cd & c^2+d^2 \\ nh & nh & n^2+h^2 \end{vmatrix} = 2\times 0 = 0$$

Q(33) $\begin{vmatrix} 1 & i & \omega \\ \omega i & -\omega & 0 \\ \omega^2 i & -\omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 1 & i & \omega \\ \omega i & i^2\omega & 0 \\ \omega^2 i & i^2\omega^2 & \omega \end{vmatrix}$ i common factor from C_2

$$i\begin{vmatrix} 1 & 1 & \omega \\ \omega i & i\omega & 0 \\ \omega^2 i & i\omega^2 & \omega \end{vmatrix} = i\times 0 = 0$$

Q(34)

$$\begin{vmatrix} 4 & 3 & 2K \\ 1 & k & 0 \\ 3 & 2 & 2K \end{vmatrix} = 0 \quad \therefore 2K\begin{vmatrix} 4 & 3 & 1 \\ 1 & k & 0 \\ 3 & 2 & 1 \end{vmatrix} = 0 \quad \therefore R_1 - R_3 \;\therefore 2K\begin{vmatrix} 1 & 1 & 0 \\ 1 & k & 0 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$2K(K-1)=0 \quad \therefore K=0 \;\; or \;\; k=1$

Q(35)

$$\begin{vmatrix} X+Y+Z+2 & Y & Z \\ X+Y+Z+2 & Y+2 & Z \\ X+Y+Z+2 & Y & Z+2 \end{vmatrix} = -4 \quad \therefore R_2 - R_1 \;, R_3 - R_1$$

$$\begin{vmatrix} X+Y+Z+2 & Y & Z \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = -4 \quad \therefore 4(X+Y+Z+2) = -4 \quad \therefore \; X+Y+Z=-3$$

Q(36) $\begin{vmatrix} 3X & 3X & 3X \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix} \; R_3 + R_2 \;\therefore \begin{vmatrix} 3X & 3X & 3X \\ 1 & b & a \\ a+b+1 & a+b+1 & a+b+1 \end{vmatrix}$

$$= 3X(a+b+1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad because\, R_1 = R_3$$

Q(37) $R.H.S = \begin{vmatrix} 1 & -X & 0 \\ X & 1 & X \\ 1 & -1 & X+1 \end{vmatrix} \; C_2 - XC_1 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ X & X^2+1 & X \\ 1 & X-1 & X+1 \end{vmatrix} \; C_2 - XC_3 \Rightarrow$

$$\begin{vmatrix} 1 & 0 & 0 \\ X & 1 & X \\ 1 & -X^2 & X+1 \end{vmatrix} \; C_3 - XC_2 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ X & 1 & 0 \\ 1 & -X^2-1 & X^3+2X+1 \end{vmatrix} = X^3+2X+1$$

$L.H.S = \begin{vmatrix} X^2 & 1 \\ -X & X \end{vmatrix} = X^3 + X \quad \therefore X^3+2X+1 = X^3+X \quad \therefore X=-1$

Q(38) $C_1 + C_2 + C_3$

$$\begin{vmatrix} X+2a & a & a \\ X+2a & X & a \\ X+2a & a & X \end{vmatrix} = (X+2a)\begin{vmatrix} 1 & a & a \\ 1 & X & a \\ 1 & a & X \end{vmatrix} = (X+2a)\begin{vmatrix} 1 & a & a \\ 0 & X-a & 0 \\ 0 & 0 & X-a \end{vmatrix}$$

$$= (X+2a)(X-a)^2$$

Q(39)

$$\begin{vmatrix} a & b & b \\ b & a+b & a \\ c & a+b+c & c \end{vmatrix} + \begin{vmatrix} a & b & 0 \\ b & a+b & b \\ c & a+b+c & a+b \end{vmatrix} = \begin{vmatrix} a & b & b \\ b & a+b & a+b \\ c & a+b+c & a+b+c \end{vmatrix} = 0$$

Q(40) $\begin{vmatrix} 1 & 1 & 1 \\ 1+Y & 1 & 1 \\ 1 & 1+Y & 1 \end{vmatrix}$ $R_1 - R_2 \Rightarrow \therefore \begin{vmatrix} -Y & 0 & 0 \\ 1+Y & 1 & 1 \\ 1 & 1+Y & 1 \end{vmatrix}$ $R_1 - R_2$

$$\Rightarrow \begin{vmatrix} -Y & 0 & 0 \\ 1 & -Y & 0 \\ 1 & 1+Y & 1 \end{vmatrix} = Y^2$$

Q(41) $A = \begin{pmatrix} 2 & 2 & -1 \\ 3 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ $\therefore |A| = -10$

$cofactors\ matrix = \begin{pmatrix} 2 & -6 & 2 \\ -5 & 5 & 0 \\ 1 & -3 & -4 \end{pmatrix}$ adjoint matrix $\begin{pmatrix} 2 & -5 & 1 \\ -6 & 5 & -3 \\ 2 & 0 & -4 \end{pmatrix}$

$$A^{-1} = \frac{1}{-10}\begin{pmatrix} 2 & -5 & 1 \\ -6 & 5 & -3 \\ 2 & 0 & -4 \end{pmatrix}$$

$$\therefore \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \frac{-1}{5} & \frac{1}{2} & \frac{-1}{10} \\ \frac{3}{5} & \frac{-1}{2} & \frac{3}{10} \\ \frac{-1}{5} & 0 & \frac{2}{5} \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Q(42)

$$|A| = \begin{vmatrix} 3 & 1 & 1 \\ K & 4 & 10 \\ 1 & 7 & 17 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ K-30 & -6 & 10 \\ -50 & -10 & 17 \end{vmatrix} = \begin{vmatrix} K-30 & -6 \\ -5 & -10 \end{vmatrix} = -10K$$

$If\ K \neq 0 \Rightarrow |A| \neq 0 \quad \therefore R(A) = 0$

$If\ K = 0$

Q(43)

$A = \begin{pmatrix} 2 & 3 & 5 \\ 7 & 4 & -2 \\ 6 & 9 & 15 \end{pmatrix}$ $square\ matrix\ of\ order\ 3 \times 3$

$|A| = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 4 & -2 \\ 6 & 9 & 15 \end{vmatrix} = 3 \begin{vmatrix} 2 & 3 & 5 \\ 7 & 4 & -2 \\ 2 & 3 & 5 \end{vmatrix} = 3 \times 0 = 0$

$\because R(A) < number\ of\ unknows$ $\therefore$ the system has infinite number off solution

$$\therefore \begin{cases} 2X + 3Y + 5Z = 0 \longrightarrow (1) \\ 7X + 4Y - 2Z = 0 \longrightarrow (2) \\ 6X + 9Y + 15Z = 0 \longrightarrow (3) \end{cases}$$

$S.S = \{-7Z, -3Z, Z\}$
