

Choose the correct answer:

Derivative:

- ① If $y = \csc(\pi - 2x)$, then $\frac{dy}{dx} = \dots\dots$
 (a) $2 \csc 2x \cot 2x$ (b) $4 \csc(\pi - 2x) \cot 2x$
 (c) $2 \csc(\pi - 2x) \cot(\pi - 2x)$ (d) $-2 \csc(\pi - 2x) \cot(\pi - 2x)$

- ② If $y = (\csc x + \cot x)^{-1}$, then $\frac{dy}{dx} = \dots\dots$
 (a) $\frac{-\csc x}{(\csc x + \cot x)^2}$ (b) $\frac{-\csc x}{\csc x + \cot x}$ (c) $\frac{\csc x}{\csc x + \cot x}$ (d) $\frac{\csc x}{(\csc x + \cot x)^2}$

- ③ If $y = 4\sec^2 x$, then $y'(\frac{\pi}{4}) = \dots\dots$
 (a) -8 (b) zero (c) $4\sqrt{2}$ (d) 16

- ④ If $f(x) = \cot x$, then $f''(\frac{\pi}{4}) = \dots\dots$
 (a) $-\frac{4}{9}$ (b) $\frac{4}{9}$ (c) 4 (d) $\frac{9}{2}$

- ⑤ If $y = \cos x$, then $\frac{d^{2018}y}{dx^{2018}} = \dots\dots$
 (a) $\cos x$ (b) $-\cos x$ (c) $\sin x$ (d) $-\sin x$

- ⑥ If $f(x) = \ln(2 + \sqrt{2}\csc x)$ where $0 < x < \frac{\pi}{2}$, then $f'(\frac{\pi}{4}) = \dots\dots$
 (a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 4

- ⑦ If $y = e^x \cos x^2$, then $\frac{dy}{dx} = \dots\dots$
 (a) $-e^x \sin x^2$ (b) $e^x (\cos x^2 - 2x \sin x^2)$
 (c) $e^x \cos x^2 - 2x \sin x^2$ (d) $-2xe^x \sin x$

- ⑧ If $x = t^3 - t$, $y = \sqrt{3t + 1}$, then $\frac{dy}{dx} = \dots\dots$ at $t = 1$
 (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{3}{4}$ (d) 8

- ⑨ If $e^{f(x)} = x^2 + 1$, then $f'(x) = \dots\dots$
 (a) $\frac{1}{x^2+1}$ (b) $\frac{2x}{x^2+1}$ (c) $2x(x^2 + 1)$ (d) $2xe^{x^2+1}$

- ⑩ If $f = 10^{(x^2-1)}$, then $\frac{dy}{dx} = \dots\dots$
- (a) $10^{(x^2-1)} \times \ln 10$ (b) $2x \times 10^{(x^2-1)}$
 (c) $2x \times 10^{(x^2-2)}$ (d) $2x(\ln 10) \times 10^{(x^2-1)}$
-
- ⑪ If $f(x) = x \sin x$, then $f''(x) + f(x) = \dots\dots$
- (a) $2 \cos x$ (b) zero (c) $-\sin x$ (d) $2x \sin x$
-
- ⑫ If $x = 5 + \sec^2 3\theta$, $y = 1 - \tan 3\theta$, then $\frac{dy}{dx} = \dots\dots$ at $\theta = \frac{\pi}{4}$
- (a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
-
- ⑬ If $x = 5 + \sec^2 3\theta$, $y = 1 - \tan 3\theta$, then the implicit relation between x and y is $\dots\dots$
- (a) $x^2 = y + 5$ (b) $x^2 + y^2 = 5$
 (c) $x^2 + y^2 = 6$ (d) $x - 6 = (1 - y)^2$
-
- ⑭ If $(x + y)^8 = 7$, then $\frac{d^2y}{dx^2} = \dots\dots$
- (a) 1 (b) zero (c) $56(x + y)^7$ (d) -1
-
- ⑮ If $\frac{dy}{dx} = 2x - 3$, $\frac{dz}{dx} = x^2 - 1$, then $\frac{d^2y}{dz^2} = \dots\dots$ at $x = 2$
- (a) $\frac{2}{27}$ (b) $\frac{2}{9}$ (c) zero (d) $-\frac{2}{9}$
-
- ⑯ If $y = \ln(x + 2)$, then $\frac{d^{2018}y}{dx^{2018}} = \dots\dots$
- (a) $\frac{2017}{(x+2)^{2018}}$ (b) $\frac{-2017}{(x+2)^{2018}}$ (c) $\frac{2018}{(x+2)^{2018}}$ (d) $\frac{-2018}{(x+2)^{2018}}$
-
- ⑰ If $f(x) = \ln \sin x - \ln \cos x$, then $\lim_{x \rightarrow \frac{\pi}{4}} \frac{f(x) - f(\frac{\pi}{4})}{x - \frac{\pi}{4}} = \dots\dots$
- (a) 1 (b) 2 (c) zero (d) -2
-
- ⑱ If $x \sin 2y = y \cos 2x$, then $\frac{dy}{dx} = \dots\dots$ at the point $(\frac{\pi}{4}, \frac{\pi}{2})$
- (a) zero (b) 1 (c) 2 (d) 3
-

- ⑱ If $f(x) = x^2$, $k(2) = 3$, $k'(2) = -2$ and $k''(2) = 5$, then $(kof)''(2) = \dots$
 (a) 38 (b) -38 (c) 10 (d) 3

-----□

- ⑳ If $y = f(x)$ an odd function and $f'(k) = m$, then $f'(-k) = \dots\dots$
 (a) m (b) -m (c) zero (d) undefined□

LIMITS:

① $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{1}{3x}} = \dots\dots$

- (a) 1 (b) 2 (c) e (d) e^2

② $\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{3x}} = \dots\dots$

- (a) $\frac{1}{3}$ (b) e^3 (c) $\sqrt[3]{e}$ (d) $\frac{e}{3}$

③ $\lim_{x \rightarrow 0} \frac{2^x - 1}{3x} = \dots\dots$

- (a) $3 \ln 2$ (b) $\frac{1}{3} \ln 2$ (c) $\frac{1}{3} \log_2 e$ (d) $2 \ln 3$

④ $\lim_{x \rightarrow 1} \frac{\log_5 x}{x-1} = \dots\dots$

- (a) $\frac{1}{\log_5 e}$ (b) 1 (c) e (d) $\log_5 e$

⑤ $\lim_{x \rightarrow 0} \frac{\ln(1+2x)}{3^x - 1} = \dots\dots$

- (a) $\frac{2}{3}$ (b) $\log_3 e^2$ (c) $\frac{3}{2}$ (d) $\frac{1}{2} \log_3 e$

⑥ $\lim_{x \rightarrow e} \frac{\ln x - 1}{x - e} = \dots\dots$

- (a) 1 (b) e (c) $\frac{1}{e}$ (d) -e

⑦ $\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}} = \dots\dots$

- (a) 1 (b) zero (c) $\frac{1}{e}$ (d) e
-

⑧ $\lim_{x \rightarrow 0} (1 + 5x)^{\frac{-2}{x}} = \dots\dots$

- (a) e^{-10} (b) $-10e$ (c) $-10 \log e$ (d) -10
-

GEOMETRICAL APPLICATIONS

- ① If the slope of the tangent to the curve $y = f(x)$ at a point $= \frac{1}{2}$ and the x-coordinate of this point decreases at a rate of 3 units /sec, then the rate of change of its y-coordinates equals $\dots\dots$ units /sec.

- (a) $\frac{-1}{6}$ (b) $\frac{-3}{2}$ (c) $\frac{1}{6}$ (d) $\frac{3}{2}$
-

- ② The ratio between the slope of the curve of the function $y = \ln 3\sqrt{x+1}$ and the slope of the curve of the function $y = \ln 5\sqrt{x+1}$ at $x = a$ is $\dots\dots$

- (a) 3 : 5 (b) 5 : 3 (c) 1 : 1 (d) $\ln 3 : \ln 5$
-

- ③ The equations of the tangent to the curve of the function: $f(x) = e^{2x+1}$ at the point $(\frac{-1}{2}, 1)$ is $\dots\dots$

- (a) $2y = x + 1$ (b) $y = 2x + 2$ (c) $y = 2x - 3$ (d) $2y = 3x + 1$
-

- ④ The slope of the tangent to the curve of the function $y = \ln(\frac{1}{2}x)$ at $x = 4$ is $\dots\dots$

- (a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 4
-

- ⑤ The equations of the inflection tangent to the curve of the function: $f(x) = x^3 + 3x^2 + 2$ is $\dots\dots$

- (a) $y = -6x - 6$ (b) $y = -3x + 1$
(c) $y = 2x + 10$ (d) $y = 3x - 1$
-

- ⑥ If the straight line $y + x = k$ is a tangent to the curve of the Function $y = x^2 + 3x + 1$, then $k = \dots\dots$

- (a) -3 (b) -2 (c) -1 (d) zero
-

- ⑦ If $y = \ln(x^2 + y^2)$, then the slope of the tangent to the curve at the point $(1, 0)$ equals $\dots\dots$

- (a) zero (b) 1 (c) $\frac{1}{2}$ (d) 2
-

⑧ If $\cos \sqrt{\pi y} = 3x + 1$, then the slope of the tangent to the curve at the point $\left(\frac{-1}{3}, \frac{\pi}{4}\right)$ equals

- (a) 3 (b) -3 (c) zero (d) 1
-

⑨ If $\lim_{z \rightarrow 0} \frac{f(z) - f(1)}{z-1} = -1$, then the angle measure of the slope of the tangent to the curve f at the point $(1, f(1))$ equals

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$
-

⑩ The tangent to the curve of the function $y + x = e^{xy}$ is vertically at the point.....

- (a) (1, 1) (b) (0, 1) only (c) (1, 0) only (d) (0, 1), (1, 0)
-

RELATED TIME RATES:

① A cube of ice melt preserving its shape by rate $1 \text{ cm}^3 / \text{sec}$, Then the rate of change of its edge length when its volume 8 cm^3 is cm / sec .

- (a) $\frac{-1}{12}$ (b) $\frac{1}{12}$ (c) $\frac{-1}{6}$ (d) $\frac{1}{6}$
-

② A body moves on the curve $y^2 = x^3$, if $\frac{dx}{dt} = \frac{1}{2}$ unit/sec at $y = -1$ then $\frac{dy}{dt}$ at this moment equals

- (a) $\frac{-3}{4}$ (b) $\frac{-3}{8}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$
-

③ A right circular cone, if both the length of its base radius and its height is increased by $\frac{1}{2} \text{ cm} / \text{sec}$ and at for a moment the length the radius of the base equals 6 cm and the height is equal to 9 cm, then the rate of change of the cone volume at that moment = cm^3 / sec

- (a) $\frac{1}{2}\pi$ (b) 10π (c) 24π (d) 54π
-

④ Circle of perimeter $P \text{ cm}$, if the radius length decreases at a rate $0.1 \text{ cm} / \text{sec}$, then the rate of change of its area = cm^2 / sec .

- (a) $-0.2P$ (b) $-0.1P$ (c) $0.1P$ (d) $(0.1)^2 P$
-

- ⑤ An empty container, its volume 9 cm^3 , water is poured in it at a rate $5 \text{ cm}^3 / \text{sec}$, the container becomes full after second.
 (a) 9 (b) 225 (c) 18 (d) 6

- ⑥ A triangle, the length of its base B is increased by $3 \text{ cm} / \text{sec}$ while its height H decreases by $3 \text{ cm} / \text{sec}$, and its surface area is A , then the phrase that is certain to be correct is the following:
 (a) A is always increasing.
 (b) A is always decreasing.
 (c) A is only decreasing when $B > H$
 (d) A is only decreasing when $B < H$

- ⑦ If the perimeter of lamina in a square-form increases by $0.4 \text{ cm} / \text{sec}$ and its surface area increases by $6 \text{ cm}^2 / \text{sec}$, then, the length of the lamina edge at that moment equals cm
 (a) 30 (b) 50 (c) 40 (d) 60

- ⑧ If the rate of increase the diameter of a balloon is equal to $1 \text{ cm} / \text{min}$ when the length of diameter is 4 cm , then the rate of change in its volume at that moment equals cm^3 / sec
 (a) 2π (b) 8π (c) 16π (d) 4π

- ⑨ A point moves on the curve $y = x^2 - 3x$. If the speed of its x -coordinate is equal to the speed of its y -coordinate, the slope of the tangent to the curve at that point equals
 (a) 1 (b) 2 (c) 3 (d) 4

BEHAVIOR OF THE FUNCTION:

- ① If the function $F(x) = x^3 + kx^2 + 4$ has an inflection points at $x = 2$ then the value of $k = \dots$
 (a) -6 (b) -3 (c) 3 (d) 6

- ② If $x \in \mathbb{R}$ then the maximum value of $4x - x^2$ is
 (a) 4 (b) 8 (c) 16 (d) 32

- ③ The curve of the function $F(x) = x^3 - 3x^2 + 2$ is convex upwards when $x \in \dots$
 (a) $]-\infty, 0[$ (b) $]-\infty, 1[$ (c) $]1, 3[$ (d) $]1, \infty[$

- 4 The function $F(x) = x^3 - 3x^2 + 5$ is decreasing on the interval....
 (a) $]0, 3[$ (b) $]0, 2[$ (c) $[0, 2]$ (d) $\mathbb{R} - [0, 2]$

- 5 If $F'(x) = ax^3 - 8b$ where a, b are constants and the point $(2, 5)$ is A local maximum point the $a \times b \in \dots$
 (a) $]2, \infty[$ (b) $]0, \infty[$ (c) $] -\infty, 0[$ (d) $]8, \infty[$

- 6 If $x, y \in \mathbb{R}^+$ where $x + y = k$ then xy is maximum when
 (a) $x = ky$ (b) $y = kx$ (c) $x = y$ (d) $xy = 1$

- 7 If $F(x)$ is a continuous function on \mathbb{R} then the true statement in each of the following is
 (a) $(a, F(a))$ is a critical point if $F'(a) = 0$
 (b) $(a, F(a))$ is an inflection point if $F''(a) = 0$
 (c) if $F'(a) = 0$ then $F(a)$ is a local maximum value.
 (d) if $F'(a) = 0$ and $F''(a) > 0$ then $F(a)$ is a local minimum value.

- 8 If $F(x)$ is a continuous function on \mathbb{R} then the true statement in each of the following is
 (a) $(a, F(a))$ is an inflection point if $F''(a) = 0$ or $F''(a)$ undefined.
 (b) $(a, F(a))$ is an inflection point if $F''(a) = 0$ and $F''(a^+) \times F''(a^-) < 0$.
 (c) $(a, F(a))$ is an inflection point if $F''(a)$ undefined and $F''(a^+) \times F''(a^-) < 0$
 (d) $(a, F(a))$ is an inflection point if $F'(a)$ exist and $F''(a^+) \times F''(a^-) < 0$.

- 9 If $F(x) = -x^3 + 6x^2 + 2x + 1$ then the greatest value of the slope of tangent to this curve is
 (a) 14 (b) 16 (c) 19 (d) -13

- 10 If $F(x) = \sqrt{x^2 - 16x}$ then $F(x)$ has a critical point when $x = \dots$
 (a) 8 only (b) 0, 16 only (c) 0, 8, 16 only (d) 0 only

- 11 If $F(x) = x - 2\sqrt{x}$ then $F(x)$ has a critical point when $x = \dots$
 (a) 0 only (b) 1 only (c) 0, 1 only (d) -1, 1 only

- 12 The curve of the function $F(x) = \frac{-5}{x-2}$ is convex downwards if
- (a) $x > 2$ (b) $x < 2$ (c) $x < 5$ (d) $x > 0$
-

13

x	0	1	2	3
$f''(x)$	5	0	-7	4

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true is

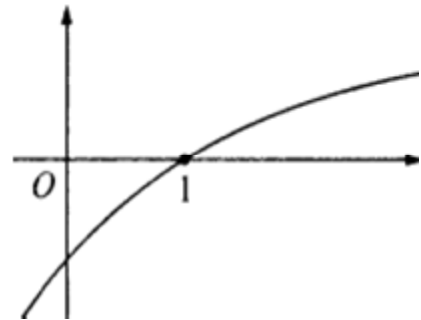
- (a) f is increasing on the interval $]0, 2[$.
 (b) f is decreasing on the interval $]0, 2[$.
 (c) The graph of f changes concavity in the interval $]0, 2[$.
 (d) f has a local maximum at $x = 1$.
-

- 14 If $F(x) = x + \frac{1}{x}$ then $F(x)$ is increasing on the interval.....

- (a) $|x| < 1$ (b) $|x| \leq 1$ (c) $|x| > 1$ (d) $|x| < 1, x \neq 0$
-

- 15 The opposite figure shows the curve of the function F which is differentiable twice at $x = 1$ the statements which must be true is

- (a) $F'(1) < F''(1) < F(1)$
 (b) $F''(1) < F'(1) < F(1)$
 (c) $F(1) < F''(1) < F'(1)$
 (d) $F''(1) < F(1) < F'(1)$
-



INTEGRATION: □

FIRST: □

- 1 $\int \tan \theta d\theta = \dots\dots$
- (a) $-\ln|\cos \theta| + c$ (b) $-\ln \cos \theta + c$
 (c) $\ln \cos \theta + c$ (d) $\ln|\cos \theta| + c$
-
- 2 $\int 4xe^{x^2} dx = \dots\dots$
- (a) $\frac{1}{2}e^{x^2} + c$ (b) $e^{x^2} + c$ (c) $2e^{x^2} + c$ (d) $4e^{x^2} + c$
-

③ $\int \frac{1}{x \ln x^3} dx = \dots\dots$

- (a) $3 \ln|x| + c$ (b) $3 \ln|\ln x| + c$ (c) $\frac{1}{3} \ln|x| + c$ (d) $\frac{1}{3} \ln|\ln x| + c$
-

④ $\int x(x^2 + 3)^5 dx = \dots\dots$

- (a) $\frac{1}{6}(x^2 + 3)^6 + c$ (b) $\frac{1}{12}(x^2 + 3)^6 + c$
 (c) $\frac{1}{4}(x^2 + 3)^4 + c$ (d) $\frac{1}{8}(x^2 + 3)^4 + c$
-

⑤ $\int \sec^4 x \tan x dx = \dots\dots$

- (a) $\frac{1}{5} \sec^5 x + c$ (b) $\frac{1}{4} \sec^4 x + c$
 (c) $\frac{1}{3} \tan^3 x + c$ (d) $\frac{-1}{3} \tan^3 x + c$
-

⑥ $\int (4 - \csc x \cot x) dx = \dots\dots$

- (a) $4x - \csc x + c$ (b) $4x + \csc x + c$
 (c) $4x - \cot x + c$ (d) $4x + \cot x + c$
-

⑦ $\int (\csc^4 x - \csc^2 x \cot^2 x) dx = \dots\dots$

- (a) $\frac{1}{5} \csc^5 x - \frac{1}{3} \cot^3 x + c$ (b) $-\tan x + c$
 (c) $-\cot x + c$ (d) $\frac{1}{3} \csc^3 x + c$
-

⑧ $\int \frac{e^x}{e^x - 3} dx = \dots\dots$

- (a) $\frac{-1}{2}(e^x - 3) + c$ (b) $-\ln|e^x - 3| + c$
 (c) $\ln|e^x - 3| + c$ (d) $\frac{1}{2} \ln|e^x - 3| + c$
-

⑨ If $\int (2x - 1)e^{2x+3} dx = yz - \int z dy$, then $\int z dy = \dots\dots$

- (a) $e^{2x+3} + c$ (b) $\frac{1}{2}e^{2x+3} + c$ (c) $-e^{2x+3} + c$ (d)
-

⑩ If $\int (2x + 3) \ln x dx = yz - \int z dy$, then $yz = \dots\dots$

- (a) $2x \ln x$ (b) $(2x + 3) \ln x$ (c) $\frac{1}{2}(2x + 3) \ln x$ (d) $x(x + 3) \ln x$
-

⑪ If $f(x) = e^{x+e^x}$, then the antiderivative for the function $f(x)$ can be $\dots\dots$

- (a) $\frac{e^{1+e^x}}{1+e^x}$ (b) $(1 + e^x)e^{e+e^x}$ (c) e^{1+e^x} (d) e^{e^x} □

- ⑫ If $\int f(x) \sin x \, dx = -f(x) \cos x + \int 3x^2 \cos x \, dx$, then $f(x) = \dots$
 (a) $3x^2$ (b) x^3 (c) $-x^3$ (d) $\sin x$
-

⑬ $\int_0^2 (2 - |x|) \, dx = \dots$

- (a) 4 (b) 2 (c) zero (d) 1
-

⑭ $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x + \cos x) \, dx = \dots$

- (a) 4 (b) 2 (c) zero (d) π
-

⑮ If $\int_{-2}^2 f(x) \, dx = 12$, $\int_{-2}^5 f(x) \, dx = 16$, then $\int_2^5 f(x) \, dx = \dots$

- (a) -28 (b) -4 (c) 4 (d) 28
-

⑯ If $f(x) = \begin{cases} x+1, & x < 0 \\ \cos \pi x, & x \geq 0 \end{cases}$, then $\int_{-1}^1 f(x) \, dx = \dots$

- (a) $\frac{1}{2} + \frac{1}{\pi}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{2} - \frac{1}{\pi}$ (d) $\frac{1}{2}$
-

⑰ If $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} \, dx = \dots$

- (a) $e - 1$ (b) $e + 1$ (c) e (d) 1
-

⑱ If $\int_{-1}^1 e^{-x^2} \, dx = k$, $\int_{-1}^0 e^{-x^2} \, dx = \dots$

- (a) k (b) $2k$ (c) $\frac{1}{2} k$ (d) $\frac{-1}{2}$
-

⑲ $\int_{-\pi}^{\pi} \frac{4x + \sin x}{x^2 + \cos x} \, dx = \dots$

- (a) $-\pi$ (b) zero (c) π (d) 2π

⑩ If we putting $y = \frac{1}{2}x$, then $\int_2^4 \frac{1 - \left(\frac{1}{2}x\right)^2}{x} dx = \dots\dots$

- (a) $\int_2^4 \frac{1-(y)^2}{y} dy$ (b) $\int_2^4 \frac{1-(y)^2}{2y} dy$ (c) $\int_1^2 \frac{1-(y)^2}{y} dy$ (d) $\int_1^2 \frac{1-(y)^2}{2y} dy$
-

SECOND:

① If $\frac{dy}{dx} = \csc^2 x$, $y = 2$ where $x = \frac{\pi}{4}$, then $y = \dots\dots$

- (a) $-(2 + \cot x)$ (b) $-(3 + \cot x)$ (c) $2 - \cot x$ (d) $3 - \cot x$
-

② If $\frac{dy}{dx} = \frac{1}{x} + x$, $y = \frac{1}{2}$ where $x = 1$, then $y = \dots\dots$ where $x = e$

- (a) $e^2 - e$ (b) $\frac{e^2-1}{2}$ (c) $\frac{e^2+1}{4}$ (d) $\frac{e^2}{2} + 1$
-

③ If the slope of the tangent to the curve of the function f at any point on it equals $\frac{1}{x-2}$ and the curve passes through the point $(3, 0)$, then $f(e^2 + 2) = \dots\dots$

- (a) 2 (b) 3 (c) $\ln 2$ (d) $\ln 3$
-

④ the volume of the solid generated by revolving the region bounded by the curve $y = \frac{1}{x}$, the two straight lines $y = 1$, $y = 2$ and y -axis complete revolution about y -axis = $\dots\dots$

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π
-

⑤ The area of the region bounded by the function curve $y = x^3$ and the two straight lines $x = -2$, $x = 2$

- (a) 1 (b) 2 (c) 4 (d) 8
-

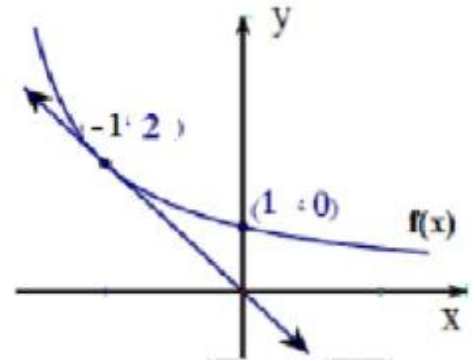
⑥ The area of the region bounded by the function curve $y = \sqrt{4 - x^2}$, and x -axis equals $\dots\dots$ square units.

- (a) 2 (b) 4 (c) 2π (d) 4
-

- ⑦ In the the opposite figure:
The curve of the function $f(x)$ has a tangent at the point $(-1, 2)$, then

$$\int_{-1}^0 x [f(x)]'' dx = \dots\dots$$

- (a) 4 (b) 3
(c) 1 (d) -1

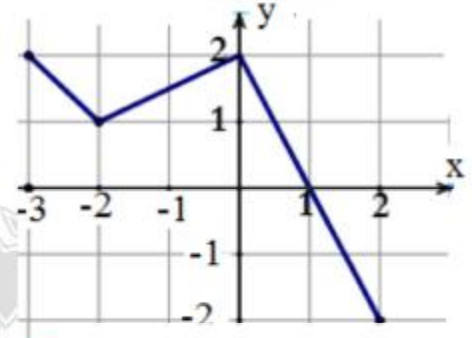


- ⑧ In the the opposite figure:
The curve of the function $f(x)$ which define by more than base and

$$g(x) = \int_{-2}^x f(t) dt$$

has a greatest value is

- (a) $f(2)$ (b) $f(-2)$
(c) $f(0)$ (d) $f(1)$

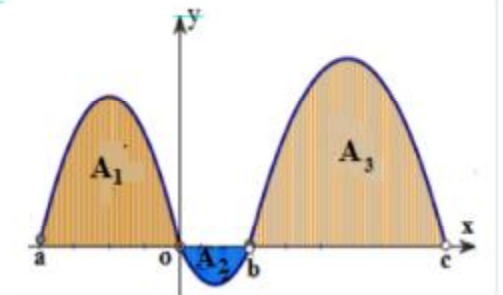


- ⑨ In the the opposite figure:

$$\int_a^c f(x) dx = 8 \int_b^0 f(x) dx \text{ and}$$

$A_1 + A_2 + A_3 = 30$ square unit, then $A_2 = \dots\dots$

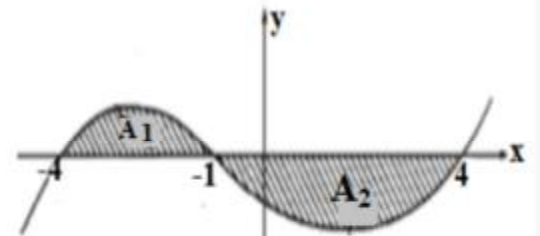
- (a) 1 square unit (b) 2 square unit
(c) 3 square unit (d) 4 square unit



- ⑩ In the the opposite figure:
The curve of the function $f(x)$ and M_1, M_2 are two +ve numers represent the two shadow regions, then

$$\int_{-4}^4 g(x) dx - 2 \int_{-1}^0 g(x) dx = \dots\dots$$

- (a) $M_1 + M_2$ (b) $M_1 - M_2$
(c) $2M_1 - M_2$ (d) $M_1 + 2M_2$



DERIVATIVE:

- (1) Find the first derivative of the function $y = x^2 \sec \frac{1}{x}$

- (2) Find the slope of tangent to the curve: $y = 2 \cot x - \sqrt{2} \sec x$
at $x = \frac{\pi}{4}$

- (3) If $y = \cos 2\pi\theta, x = \sin 2\pi\theta$, then find $\frac{dy}{dx}$ at $\theta = \frac{1}{6}$

- (4) Find the value of the parameter t as which the curve,
 $x = 2t^3 - 5t^2 + 4t + 9, y = 2t^2 + t - 5$
(a) Has vertical tangent (b) Has horizontal tangent

- (5) Using the parametric differentiation find the first derivative of
 $(x - \sin x)$ With respect to $(1 - \cos x)$ at $x = \frac{\pi}{3}$

- (6) If $y = \sqrt{2x + 5}$, then prove that $(2x + 5) \frac{d^3y}{dx^3} + 3 \frac{d^2y}{dx^2} = 0$.

- (7) If $y^2 + x^2 y^2 = 8$, then prove that $(x^2 + 1) \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 0$

- (8) If $y^2 + ax^3 - bx = c$, then prove that $yy'' + (y')^2 + 3ax = 0$

- (9) If $\sin 2x - \cos 3x = 0$, then prove that:
 $3 \frac{d^2y}{dx^2} \tan 3y + 9 \left(\frac{dy}{dx}\right)^2 - 4 = 0$.

- (10) If $xy = 1$, then prove that $\frac{d^2y}{dx^2} + 3y \frac{dy}{dx} + y^3 = 0$.

- (11) If $y = x^3 + 1, z = x^2 + 1$, then find the value of $\frac{d^2z}{dy^2}$ at $x = 2$

(12) If $x^2 = 2t - 3$, $y = 2t^2 - 1$, then prove that:

$$3 \frac{dy}{dx} - x \frac{d^2y}{dx^2} - 12x = 0$$

(13) If $x = \sin 2t$, $y = \cos 2t = 0$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.

(14) If $y = \tan^2 x$, then prove that $\frac{d^2y}{dx^2} = 2(1 + y) (1 + 3y)$

(15) If $y = xe^x$, then prove that $x \frac{dy}{dx} = y(x + 1)$

GEOMETRICAL APPLICATIONS □

(1) Find the equation of the tangent and the normal to the curve: $x^2 + y^2 - 6x - 16 = 0$ at its intersection point with $y - axis$, where $y > 0$.

(2) Find the equations of the tangent and the normal to the curve: $x = y \sec x$ at the point in which $x = \pi$.

(3) If the parametric equations of the function curve $y = f(x)$ are $x = \sec^2 \theta - 1$, $y = \tan \theta$, then find the equation of the tangent and normal at $\theta = -\frac{\pi}{4}$

(4) Prove that the two curves $y = x^2 - x + 2$, $y = 3x - x^2$ are tangential and find the equation of the common tangent.

(5) Find the points of the intersection of the two curves $xy = 2$, $x^2 - y^2 = 3$, then prove that they intersect orthogonally.

(6) Find the values of constants a, b, c such that the two curves $y = ax^3 + bx$, $y = cx^2 - x$ has a common tangent at the point $(-1, 2)$

- (7) Find the area of the triangle bounded by x-axis , tangent and normal to the curve: $3x^2 + y^2 = 12$ at the point $(- 1, 3)$ which lies on the curve.

- (8) Find the points on the curve $2y^2 - 8y + x = 0$, at which the tangent is parallel to y – axis.

- (9) Find the points on the curve $y = x^3 - x$ at which the tangent passes through the point $(1, - 4)$.

- (10) Find the points on the y - axis such that the two tangents drawn from it to the curve $4y + x^2 = 0$ with the straight line passes through the points of tangency form an equilateral triangle.

RELATED TIME RATES:

- (1) A spherical balloon is filled with gas but the gas leaks at the rate of $x \text{ cm}^3 / \text{sec}$. Prove that the rate of the decrease of the balloon area at the moment which the radius length $r \text{ cm}$ equal $\frac{2x}{r} \text{ cm}^2 / \text{sec}$.

- (2) A cube extends by heat so its edge length increases at a rate $0.02 \text{ cm} / \text{min}$. and its surface area increases at $0.72 \text{ cm}^2 / \text{min}$ at a moment. Find the cube edge length at of this moment and the rate of increase in its volume at this time.

- (3) A metal body in the form of cuboid with square base, its base side increases at a rate of $1 \text{ cm} / \text{min}$ and its height decreases at a rate $2 \text{ cm} / \text{min}$. Find the rate of increasing of its volume when its base side length 5 cm and its height 20 cm , then find after how many minutes this Increase will be vanish.

- (4) A metal regular quad pyramids whose height equals its base side length. Its volume increases at a rate of $1\text{ cm}^3/\text{sec}$, if the rate of the increase of both the pyramid's height and its base side length equals $0.01\text{ cm}/\text{sec}$ find its base side length.

- (5) ABC is a right- angled triangle at C, its area is constant and equals 24 cm^2 , if the rate of change of b equals $1\text{ cm}/\text{sec}$. find the rate of change for each of a and $m(\angle A)$, at the moment in which b equals 8 cm .

- (6) A regular octagon whose side length is 10 cm and it increases at a rate of $0.2\text{ cm}/\text{sec}$. Find the rate of increase of its area.

- (7) The point A(x, y) moves on the function curve $y = x^3 + x$ where $\frac{dy}{dt} = 2\text{ unit}/\text{sec}$. Find the rate of change of the area of the triangle A B C where O is the origin and B (0, 6) at the moment at which the X-coordinate of the moving point equals 3.

- (8) A circular segmet in which the length of radius of its circle is 10 cm and its center angle measured x° and change at a rate of $3^\circ/\text{min}$. Find the rate of increase of its area at $x = 60^\circ$.

- (9) A 5-meter long ladder is leaning by its lower end on a vertical wall and its other end on a horizontal ground, if the lower end slides a way from the wall at a rate of $\frac{1}{2}\text{ m}/\text{min}$. Then find the rate of sliding the top of the ladder, when the lower end at 3 m from the wall. Then find the lower end distance from the wall when the two sides move at the same rate.

- (10) An isosceles triangle whose base length is 8 cm and its height changes at a rate of $2\text{ cm}/\text{min}$. Find the rate of change of the vertex angle when its height is 6 cm .

- (11) A 180 cm man standing in front of a lamp rising from the surface of the ground by 5.4 meters. If the man moves away from the lamp on a horizontal road at a fixed speed of 3 m / sec, then:
- The rate of change in the length of a man's shadow.
 - The speed of the end of man's shadow.
 - The rate of change of the man's head from the lamp when the man is 4.8 m from the lamppost.
-
- (12) Water is poured into a cylindrical container at a rate of $2 \text{ cm}^3 / \text{sec}$. Find the rate of change at the height of the water rises in the container such that base radius length 2 cm. If the container height is 2.8 cm, when does the container become full. ($\pi = \frac{22}{7}$)
-

BEHAVIOR OF THE FUNCTION

- (1) Determine the increasing and decreasing intervals of the function $f(x) = \sin x + \cos x$ where $x \in [0, \frac{3\pi}{2}]$.
-
- (2) Determine the increasing and decreasing intervals of the function $f(x) = x - e^x$.
-
- (3) Determine the increasing and decreasing intervals of the function $f(x) = x + \ln x$.
-
- (4) Find the local maximum and minimum values (if existed), of the $f(x) = x^3 - 9x^2 + 24x + 10$ and show its type.
-
- (5) Find the local maximum and minimum values (if existed), of the $f(x) = \sqrt[3]{(x-1)^2} + 2$ and show its type.
-
- (6) Find the local maximum and minimum values (if existed), of the $f(x) = \frac{x^2}{1-x} + 2$ and show its type.
-

- (7) Find the local maximum and minimum values(if existed), of the $f(x) = \ln(e^{2x} - 2e^x + 3)$ and show its type.

- (8) Find the local maximum and minimum values(if existed), of the $f(x) = e^{\frac{1}{x}} \left(\frac{1}{x} - 2 \right)$ and show its type.

- (9) Find the local maximum and minimum values(if existed), of the $f(x) = x^2 - 1 - 2 \ln|x|$ and show its type.

- (10) Determine the intervals of convexity upwards, downwards and The inflection points (if existed)of the function curve:
 $f(x) = x^3 + 2x^2 - 4x - 8$.

- (11) If the function f where $f(x) = x^3 + ax^2 + bx$ has an inflection point at (2 , 2), then find the values of a , b.

- (12) Find the extrema values of the following functions on the given interval:
 (a) $f(x) = 1 + 12x - x^3$ [1 , 3]
 (b) $f(x) = x^2 e^x$ [-3,1]

- (13) Sketch the curve of the function $f(x) = 2 + 3x - x^3$

APPLICATIONS OF MAXIMUMA AND MINIMUMA

- (1) A right- angled triangle whose hypotenuse length is 26 cm. Find the length of the two legs of the right angle,if the length of the altitude from the right angle on the hypotenuse is as maximum as possible.

- (2) If the straight line L intersect the coordinates axes at the points A , B and passes throw the point C = (8, 1). Find the smallest length of piece \overline{AB} .

- (3) Determine the largest area of a rectangular region drawn in a ΔABC with two sides applied to the coordinate axes where $A = (3, 0)$, $B = (0, 4)$, and $O = (0, 0)$.
-
- (4) \overline{AB} is a diameter in a circle M , $E \in$ the circle. From E the tangent is drawn to the circle and intersect the two tangent are drawn to the circle from A, B at C, D respectively. Prove that the minimum area of the trapezium $ABCD$ equals double square length diameter of the circle.
-
- (5) Find the dimensions of the largest rectangle can be drawn in the isosceles triangle, the length of its base 18 cm, the height 12 cm such that two vertices lie on the triangle base and the other two vertices on each side of the triangle.
-
- (6) A rectangular Parallelepiped has a volume 576 cm^3 and the ratio between the two lengths of its base is $2 : 1$. Find the dimensions of the parallelepiped that makes its total surface area minimum.

INTEGRATION:**FIRST:** Find each of the following:

①	$\int (1 + \tan^2 x) \cos^2 x \, dx$	⑪	$\int x^3 \sqrt{4 - x^2} \, dx$
②	$\int \frac{dx}{\sqrt{x}(1 + \sqrt{x})^3}$	⑫	$\int (3 + \sin x)^5 \cos x \, dx$
③	$\int (x^2 + 5) \sqrt{x - 1} \, dx$	⑬	$\int \frac{\sec^2(\ln x)}{x} \, dx$
④	$\int x e^{-2x} \, dx$	⑭	$\int x^2 \sqrt[3]{3x + 1} \, dx$
⑤	$\int \frac{\ln x}{\sqrt{x}} \, dx$	⑮	$\int \ln(x + 1) \, dx$

⑥	$\int x \sin x \, dx$	⑩	$\int x^3 \ln x \, dx$
⑦	$\int \frac{4x}{\sqrt[3]{2x+1}} \, dx$	⑪	$\int x \sec^2 x \, dx$
⑧	$\int e^{\sqrt{x}} \, dx$	⑫	$\int \frac{3x+5}{e^{2x}} \, dx$
⑨	$\int \frac{\ln x}{x^2} \, dx$	⑬	$\int \frac{x e^x}{(x+1)^2} \, dx$
⑩	$I = \int x (\ln x)^2 \, dx$	⑭	$\int_{-3}^3 x^2 - 4 x \, dx$

SECOND: application of integration:

(1) If $\int_3^{a-1} f(x) \, dx = \int_2^3 2f(2x-1) \, dx$. Then find the value of a.

(2) Find the equation of the curve passing through the point $(\frac{\pi}{2}, \frac{\pi^2}{4} + 9)$ if its slope of tangent at any point is (x, y) on it given in relation $m = 2x + \frac{1}{2} \sec^2 \frac{x}{2}$

(3) Find the equation of the curve passing through the two points $(\frac{\pi}{4}, 5)$ $(\frac{3\pi}{4}, 1)$ if its slope of tangent at any point is (x, y) on it given in relation $m = -a \csc^2 x$, a constant.

(4) If the slope of the normal to a curve at any point on it (x, y) is given by relation $m = \csc x \sec x$, find the equation of the curve, note that it passes through the point $(\frac{\pi}{6}, 1)$

- (5) The slope of the tangent to a curve at any point (x, y) on it is given by the relation $\frac{dy}{dx} = \frac{3}{y+\sqrt{y}}$, find the equation of the curve, note that it passes through the point $(\frac{1}{18}, 1)$.
-

- (6) If the function curve $y = f(x)$ has a local maximum value at the point $(2, 7)$ and $\frac{d^2y}{dx^2} = 2 - 6x$. find the equation of the curve.
-

- (7) If $y \frac{dy}{dx} + 2x = 3$ and the curve passes through the point $(1, 2)$. Then find the relationship between x, y .
-

- (8) If the rate of change of the slope of the tangent to a curve at any point on it is $6x - 3$. find the equation of the curve, note that it passes through the point $(2, 2)$ and the tangent at $x = 1$ is horizontal.
-

- (9) If the slope of the tangent to a curve at any point is given by the relation $\frac{dy}{dx} = 3x^2 - 18x + 24$ and the curve has a local minimum value equal 26. Find the local value maximum of the function.
-

- (10) Find the area of the region bounded by the function curve $f(x) = x^3$, the x - axis and the two straight lines $x = -2, x = 2$.
-

- (11) If $f:] - \infty, 3] \rightarrow \mathbb{R}$ where $f(x) = x^3 - 4x$. find the area of the region above x - axis bounded by the function curve and x - axis.

(12) Find the area of the region bounded by the function curve
 $f(x) = 3 + 2x - x^2$ and the straight lines $x = -1$, $x = 4$ and $y = 0$.

(13) Find the area of the region bounded by the two curves
 $f(x) = x^3 - 3x^2 + 5$, $(x) = x + 2$.

(14) If the cost of a squared metre of granite to cover the floor of a hotel corridors is L.E.400 and five corridors have been already covered with granite and the area of each is bounded by the curve of the function f and the two straight lines $x = 0$, $y = 0$, where
 $f(x) = 12 - \frac{1}{3}x^2$. find the cost covering the five corridors.

(15) Find the volume of the solid generated by revolving the region bounded by the two curves $y = \sqrt{x}$, $y = x^2$ a complete revolution about $x - axis$.

(16) Find the volume of the solid generated by revolving the region bounded by the two curves $y = 4 - x^2$, $2x + y = 4$ a complete revolution about $y - axis$.

(17) Find the volume of the solid generated by revolving the region bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x - axis$ where a and b are constants a complete revolution about $x-axis$

Answers of Choose**Deivatives:**

(1)	c	(2)	c	(3)	d	(4)	c	(5)	b
(6)	b	(7)	b	(8)	b	(9)	b	(10)	d
(11)	a	(12)	c	(13)	d	(14)	b	(15)	a
(16)	a	(17)	b	(18)	c	(19)	a	(20)	a

Limits:

(1)	a	(2)	c	(3)	b	(4)	d	(5)	b
(6)	c	(7)	d	(8)	d				

Geometrical applections:

(1)	b	(2)	c	(3)	b	(4)	b	(5)	b
(6)	a	(7)	d	(8)	b	(9)	d	(10)	c

Related tim rates:

(1)	a	(2)	a	(3)	c	(4)	b	(5)	d
(6)	c	(7)	a	(8)	b	(9)	a		

Behavior of the Function

(1)	a	(2)	a	(3)	b	(4)	b	(5)	b
(6)	c	(7)	d	(8)	d	(9)	a	(10)	b
(11)	c	(12)	b	(13)	c	(14)	c	(15)	d

Integration: (first)

(1)	a	(2)	c	(3)	d	(4)	b	(5)	b
(6)	b	(7)	c	(8)	c	(9)	b	(10)	d
(11)	d	(12)	b	(13)	b	(14)	b	(15)	c
(16)	d	(17)	a	(18)	c	(19)	b	(20)	c

Integration: (second)

(1)	d	(2)	d	(3)	a	(4)	b	(5)	c
(6)	c	(7)	d	(8)	d	(9)	c	(10)	a

Producing answers questions**Deivatives:**

(1)		(2)	-2
-----	--	-----	----

(3)	$-\sqrt{3}$	(4)	$t = \frac{2}{3}, t = 1, t = \frac{-1}{4}$
(5)	$\frac{\sqrt{3}}{3}$	(6)	prove
(7)	prove	(8)	prove
(9)	prove	(10)	prove
(11)	$\frac{-1}{72}$	(12)	prove
(13)	1	(14)	prove
(15)	prove		

Geometrical applications:

(1)	equation of the tangent is $y = 8$ equation of the normal is $x = 0$	(2)	equation of the tangent is $y + x = 0$ equation of the normal is $x - y - 2\pi = 0$
(3)	equation of the tangent is $y + x = 0$ equation of the normal is $x - y - 2 = 0$	(4)	equation of the tangent is $x - y + 1 = 0$
(5)	prove	(6)	$a = \frac{-1}{2}, b = \frac{-3}{2}$ and $c = 1$
(7)	The area = 9 square unit	(8)	The point is (8, 2)
(9)	The points are (3, 6), (-1, 2)	(10)	The point is (0, 2)

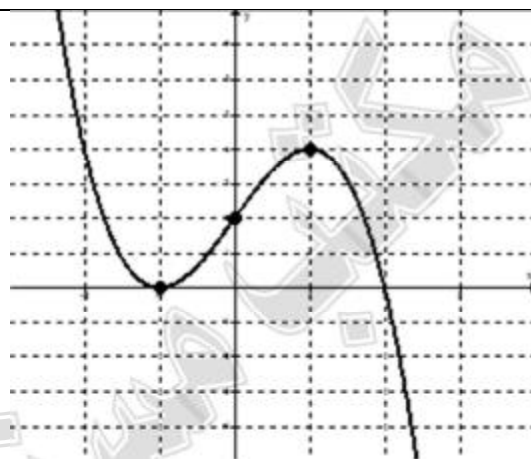
Related tim rates:

(1)	$-4 \text{ cm}^2/\text{sec}$	(2)	$0.54 \text{ cm}^3/\text{min}$
(3)	$150 \text{ cm}^3/\text{min}$, 3 min.	(4)	10 cm
(5)	$\frac{-3}{4}, \frac{-3}{25}$	(6)	19.3
(7)	$\frac{3}{28}$	(8)	$75 \text{ cm}^2/\text{min}$
(9)	$\frac{-3}{8}, 2.5\sqrt{2}$	(10)	$\frac{-4}{13}$
(11)	1.5, 4.5, 2.4	(12)	$\frac{7}{44}, 17.6$

BEHAVIOR OF THE FUNCTION

1	The function is increasing $\forall x \in \left]0, \frac{\pi}{4}\right[\cup \left]\frac{3\pi}{4}, \frac{5\pi}{4}\right[$ and decreasing $\forall x \in \left]\frac{\pi}{4}, \frac{5\pi}{4}\right[$
2	The function is increasing $\forall x \in]0, \infty[$ and decreasing $\forall x \in]-\infty, 0[$
3	The function is increasing on its domain.
4	At $x = 2$ there is a local maximum value = 30, and (2, 30) is a maximum point. At $x = 4$ there is a local minimum value = 26, and (4, 26) is a minimum point.
5	At $x = 1$ there is a local minimum value = 2, and (1, 2) is a minimum point.
6	At $x = 2$ there is a local maximum value = -2, and (2, -2) is a maximum point. At $x = 0$ there is a local minimum value = 2, and (0, 2) is a minimum point.
7	At $x = 0$ there is a local minimum value ≈ 0.69 , and (0, 0.69) is a minimum point.
8	At $x = 1$ there is a local minimum value = -e, and (1, -e) is a minimum point.

9	At $x = 1$ there is a local minimum value $= -e$, and $(1, -e)$ is a minimum point	
10	The curve is convex upwards $\forall x \in]-\infty, \frac{-2}{3}[$, downwards $\forall x \in]\frac{-2}{3}, \infty[$ And $(\frac{-2}{3}, \frac{-128}{27})$ is inflection point.	
11	$A = -6, b = 9$	
12	(i) The absolute maximum value $= 17$ at $x = 2$, the absolute minimum value $= 10$ at $x = 3$ (ii) The absolute maximum value $= e$ at $x = 1$, the absolute minimum value $= 0$ at $x = 0$	
13	point	Its kind
	$(1, 4)$	maximum
	$(-1, 0)$	minimum
	$(0, 2)$	inflection
	$(2, 0)$	On x-axis
	$(-2, 4)$	assistant



APPLICATIONS OF MAXIMUMA AND MINIMUMA

1	$13\sqrt{2}, 13\sqrt{2}cm$	4	prove
2	$5\sqrt{5}$ henght unit	5	6 cm , 9 cm
3	3 square unit	6	6, 12 and 8cm

INTEGRATION

FIRST

1	$x + c$
2	$-(1 + \sqrt{x})^{-2} + c$
3	$\frac{2}{35} \sqrt{(x-1)^3(5x^2 + 4x + 61)} + c$
4	$\frac{-1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + c$
5	$4\sqrt{x} \ln \sqrt{x} - 4\sqrt{x} + c$
6	$x \cos x - \sin x + c$
7	$\frac{3}{5} (2x + 1)^{\frac{5}{3}} - \frac{3}{2} (2x + 1)^{\frac{2}{3}} + c$
8	$2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
9	$\frac{-1}{x} \ln x - x^{-1} + c$

10	$\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + c$
11	$\frac{-1}{3}x^2(4-x^2)^{\frac{3}{2}} - \frac{2}{15}(4-x^2)^{\frac{5}{2}} + c$
12	$\frac{1}{6}(3 + \sin x)^6 + c$
13	$\tan \ln x + c$
14	$\frac{1}{27} \left[\frac{3}{10}(3x+1)^{\frac{10}{3}} - \frac{6}{7}(3x+1)^{\frac{7}{3}} + \frac{3}{4}(3x+1)^{\frac{4}{3}} \right] + c$
15	$(x+1) \ln(x+1) - x + c$
16	$\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + c$
17	$x \tan x + \ln \cos x + c$
18	$\frac{-1}{2}(3x+5)e^{-2x} - \frac{3}{4}e^{-2x} + c$
19	$\frac{e^x}{x+1} + c$
20	$\frac{46}{3}$

SECOND	
1	5
2	$y = x^2 + \tan \frac{x}{2} + 10$
3	$y = 2 \cot x + 3$
4	$y = \sin^2 x + \frac{3}{4}$
5	$3y^2 + \sqrt{y^3} = 18x + 6$
6	$y = x^2 - x^3 + 8x - 5$
7	$y^2 = 6x - x^2$
8	$y^2 = x^3 - \frac{3}{2}x^2$
9	The local maximum value = 30
10	8 square unit
11	4 square unit
12	13 square unit
13	8 square unit
14	The cost of covering the five corridors = 69000 L.E.
15	$\frac{3}{10}\pi$
16	$\frac{8}{3}\pi$
17	$\frac{4}{3}\pi a b^2$