Booklets Exams
1 , 2 , 3

Calculus

2018

SEC.3

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Booklet

1

①

If $y = \cot x$, then $y''\left(\frac{\pi}{4}\right)$ equals .....

ⓐ $\frac{-4}{9}$ ⓑ $\frac{4}{9}$

● 4 ⓓ $\frac{9}{2}$

إذا كانت د(س) = ظتا س فإن د'' $\left(\frac{\pi}{4}\right)$ تساوي ......

(أ) $\frac{-٤}{٩}$ (ب) $\frac{٤}{٩}$

(ج) ٤ (د) $\frac{٩}{٢}$

$$y' = -\csc^2 x$$

$$y'' = -2\csc x(-\csc x \cot x)$$

$$= 2\csc^2 x \cot x$$

$$y''\big)_{x=\frac{\pi}{4}} = 2\csc^2\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right)$$

$$= 2(\sqrt{2})^2(1)$$

$$= 2(2)$$

$$= \boxed{4}$$

②

If $x^2 + y^2 = 1$

, then $\left(\frac{dy}{dx}\right)$ equals ......

ⓐ x

ⓑ $\frac{1}{y}$

ⓒ $\frac{-y}{x}$

● $\frac{-x}{y}$

إذا كان $\text{س}^{٢} + \text{ص}^{٢} = ١$

فإن $\frac{\text{د ص}}{\text{د س}}$ تساوي ......

(أ) س

(ب) $\frac{١}{\text{ص}}$

(جـ) $\frac{-\text{ص}}{\text{س}}$

(د) $\frac{-\text{س}}{\text{ص}}$

$x^2 + y^2 = 3$ (der. w.r.t x)

$2x + 2y\frac{dy}{dx} = 0$ (÷2)

$x + y\frac{dy}{dx} = 0$ (der. w.r.t x)

$y\frac{dy}{dx} = -x$

$\frac{dy}{dx} = \boxed{\frac{-x}{y}}$

③

(3) If $x^2 + y^2 = 3$, prove that :

$$y^3\frac{d^2y}{dx^2}+3=0$$

إذا كان س² + ص² = ٣

اثبت أن ص³ د²ص / د س² + ٣ = صفر

$$\boxed{x^2+y^2=3}\to(1) \quad (\text{der. w.r.t } x)$$

$$2x+2y\frac{dy}{dx}=0 \quad (\div 2)$$

$$\boxed{x+y\frac{dy}{dx}=0}\to(2) \quad (\text{der. w.t } x)$$

$$1+y\left(\frac{d^2y}{dx^2}\right)+\frac{dy}{dx}\left(\frac{dy}{dx}\right)=0$$

$$\boxed{1+y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0}\to(3)$$

from (2) $\quad \frac{dy}{dx}=\frac{-x}{y}$

in eq. (3) $\quad 1+y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{-x}{y}\right)^2=0$

$$1+y\left(\frac{d^2y}{dx^2}\right)+\frac{x^2}{y^2}=0 \quad (\otimes y^2)$$

$$\underline{\underline{y^2}}+y^3\left(\frac{d^2y}{dx^2}\right)+\underline{\underline{x^2}}=0$$

from (1)

$$\therefore\ y^3\left(\frac{d^2y}{dx^2}\right)+3=0 \quad \#$$

④ A constant length ladder its upper end is sliding on a vertical wall at a rate of k unit/sec
find the rate of sliding of its base away from the wall when the ladder leans with the wall by an angle θ where $\csc\theta = \frac{5}{4}$

سلم ثابت الطول ينزلق طرفه العلوي على حائط رأسي بمعدل ك وحدة طول/ث.

أوجد معدل ابتعاد طرفه السفلي عن الحائط عندما يميل السلم على الرأسي بزاوية θ حيث قتا θ = $\frac{5}{4}$

$$x^2 + y^2 = L^2 \quad \text{(dr. w.r.t x)}$$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0 \quad \div 2$$

$$x\left(\frac{dx}{dt}\right) + y\left(\frac{dy}{dt}\right) = 0$$

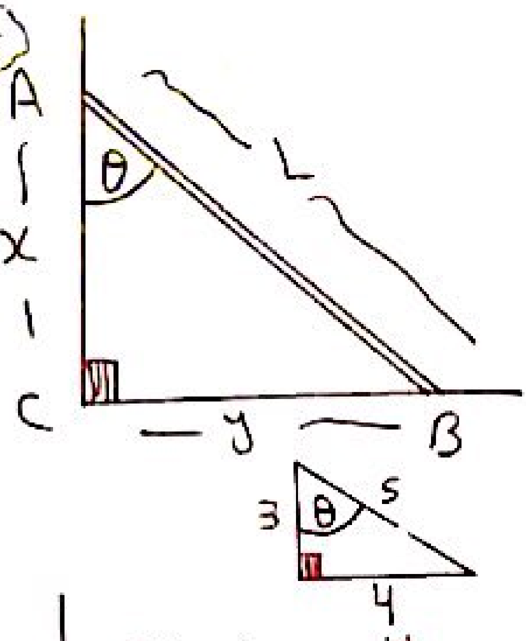
$$x(-k) + y\left(\frac{dy}{dt}\right) = 0$$

$$\frac{dy}{dt} = \frac{xk}{y}$$

$$\therefore \frac{dy}{dt} = \frac{k(K\cos\theta)}{K\sin\theta}$$

$$= \frac{k(3/5)}{(4/5)}$$

$$= \boxed{\frac{3}{4}k} \text{ unit length/sec}$$



$$\because \sin\theta = \frac{y}{L}$$

$$\therefore \boxed{y = L\sin\theta}$$

$$\because \cos\theta = \frac{x}{L}$$

$$\boxed{x = L\cos\theta}$$

⑤

$\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x}$ equals ....

ⓐ 1 ⓑ 2

ⓒ e ● e^2

$\underset{س \to \infty}{\text{نهـا}} \left(١ + \frac{١}{س}\right)^{٢س}$ تساوي ....

Ⓐ ١ ⓑ ٢

ⓙ هـ ⓓ هـ٢

Let $\frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

when $x \to \infty \Rightarrow y \to 0$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x}$$

$$= \lim_{y \to 0} (1 + y)^{\frac{2}{y}}$$

$$= \lim_{y \to 0} \left[(1 + y)^{\frac{1}{y}}\right]^2$$

$$= \boxed{e^2}$$

⑥ If $f(x) = e^{3x}$, then $f'(x) = \ldots\ldots\ldots$

ⓐ e^{3x}
● $3e^{3x}$
ⓒ $9e^{3x}$
ⓓ $3e^{2x}$

إذا كان د(س) = $\text{هـ}^{\text{٣س}}$

فإن دَ(س) = ...

(أ) $\text{هـ}^{\text{٣س}}$
(ب) ٣ $\text{هـ}^{\text{٣س}}$
(جـ) ٩ $\text{هـ}^{\text{٣س}}$
(د) ٣ $\text{هـ}^{\text{٢س}}$

$f(x) = e^{3x}$

$f'(x) = \boxed{3e^{3x}}$

⑦

$\int \left(3x^2 + \frac{5}{x}\right) dx$ equals ....

ⓐ $6x - \frac{5}{x^2} + c$

● $x^3 + 5\ln|x| + c$

ⓒ $x^3 - 5\ln x + c$

ⓓ $3x^3 + 5\ln|x| + c$

$\int \left(\text{٣س}^{٢} + \frac{\text{٥}}{\text{س}}\right) \text{ءس}$ يساوي .....

Ⓘ $\text{٦س} - \frac{\text{٥}}{\text{س}^{٢}} + \text{ث}$

(ب) $\text{س}^{٣} + \text{٥ لوهـ}|\text{س}| + \text{ث}$

(ج) $\text{س}^{٣} - \text{٥ لوهـ س} + \text{ث}$

(د) $\text{٣س}^{٣} - \text{٥ لوهـ}|\text{س}| + \text{ث}$

$$\int \left(3x^2 + \frac{5}{x}\right) dx$$

$$= \frac{3x^3}{\cancel{3}} + 5\ln|x| + C$$

$$= x^3 + 5\ln|x| + C$$

⑧ If the slope of the tangent to the curve of the function f at any point (x, y) belonging to it equals $\frac{1}{2x-e}$ and $f(e) = \frac{1}{2}$, find $f(2e)$.

إذا كان ميل المماس لمنحنى الدالة د عند أي نقطة (س ، ص) عليه يساوي $\frac{\text{١}}{\text{٢س - هـ}}$ وكان د (هـ) = $\frac{\text{١}}{\text{٢}}$ أوجد د (٢ هـ).

$$\frac{dy}{dx} = \frac{1}{2x-e}$$

$$y = \int \frac{2}{2x-e}\,dx$$

$$= \frac{1}{2}\ln|2x-e| + c$$

$$\because f(e) = \frac{1}{2}$$

$$\therefore \frac{1}{2} = \frac{1}{2}\ln|2e-e| + c$$

$$\frac{1}{2} = \frac{1}{2}\ln|e| + c$$

$$\frac{1}{2} = \frac{1}{2} + c \Rightarrow c = 0$$

$$\therefore y = \frac{1}{2}\ln|2x-e|$$

$$f(2e) = \frac{1}{2}\ln|2(2e)-e|$$

$$= \frac{1}{2}\ln|3e|$$

$$= \frac{1}{2}[\ln 3 + \ln e] = \boxed{\ln\sqrt{3} + \frac{1}{2}}$$

⑨ The function $f : f(x) = x^3 + 6x + 2$ is increasing when $x \in$ ....

ⓐ $]-6,\infty[$ ⓑ $]-\infty,-3[$

ⓒ $]-3,\infty[$ ● R

الدالة : د(س) = س^٣ + ٦س + ٢ تكون متزايدة عندما س ∈ ....

(أ)]-٦ ، ∞[(ب)]-∞ ، -٣[

(جـ)]-٣ ، ∞[(د) ح

$f'(x) = 3x^2 + 6$

$3x^2 + 6 = 0$

$x^2 = -2$ has no sol.

$\longrightarrow$

$\longleftarrow + \quad + \quad + \quad + \quad + \quad + \longrightarrow f'(x)$

increasing on R

⑩ If the curve $y = (5x - a)^3 + 4$ has an inflection point at $x = 2$, then a = ……

ⓐ 2　　ⓑ 5

ⓒ 4　　● 10

إذا كان للمنحنى ص = $(٥$ س $- أ)^٣ + ٤$

نقطة انقلاب عند س = ٢ فإن أ = ...

(أ) ٢　　(ب) ٥

(جـ) ٤　　(د) ١٠

$$y' = 3(5x - a)^2 (5)$$

$$y'' = 15(5x - a)(5)$$

$$0 = 75(5 \times 2 - a)$$

$$0 = 10 - a$$

$$\boxed{a = 10}$$

⑪ The absolute maximum value for the function $f : f(x) = -x^2$ in the interval [-3,2] is

ⓐ $f(-3)$ ● $f(0)$

ⓒ $f(1)$ ⓓ $f(2)$

القيمة العظمى المطلقة للدالة د(س) = − س٢ في الفترة [-٣ ، ٢] هي ....

أ د(-٣) ب د(٠)

جـ د(١) د د(٢)

$P'(x) = -2x$

$-2x = 0$

$\Rightarrow x = 0$

$\because 0 \in [-3, 2]$

$\therefore$ at $x=0$ there is a Critical Point

x	-3	0	2
y	-9	0	-4

absolute max

⑫ **Answer one of the following items**

(a) Determine the maximum and the minimum local values (if they exist) for the function f such that

$f(x) = 8\ln x - x^2$

(b) Determine the absolute extrema values of the function

$f: f(x) = x^3 - 3x + 2$,

$x \in [-2,1]$

أجب عن إحدى الفقرتين الآتيتين :

أ) عين القيم العظمى المحلية
والقيم الصغرى المحلية
(إن وجدت) للدالة د حيث
د(س) = ٨ لوس − س٢

ب) عين القيم القصوى
المطلقة للدالة د حيث
د(س) = س٣ − ٣س + ٢ ، س ∈ [−٢ ، ١]

a) $f(x) = 8\ln x - x^2$

$f'(x) = 8\left(\frac{1}{x}\right) - 2x$

$0 = 8\left(\frac{1}{x}\right) - 2x \quad (\times x)$

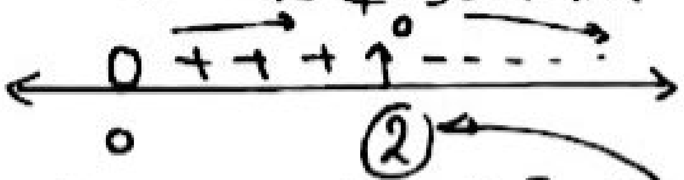
$0 = 8 - 2x^2$

$x^2 = 4 \Rightarrow x = \pm 2$

Domain of f

$f(x) :]0,\infty[$

$\therefore -2 \notin$ Domain



incr. on $]0,2[$ (l.max)

decr. on $]2,\infty[$

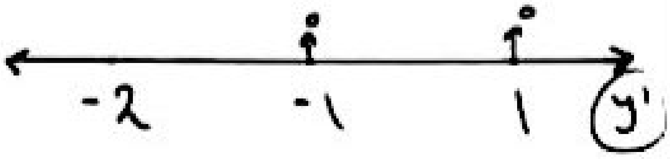
$f(2) = \boxed{8\ln 2 - 4}$ absolute max. value

b) $f'(x) = 3x^2 - 3$

$3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1 \in [-2,1]$



x	-2	-1	1
y	0	4	0

absolute min. = 0

absolute max. = 4

⑬ $\int \tan^2 x \; dx = \ldots$

● $\tan x - x + c$

ⓑ $\tan x + x + c$

ⓒ $\sec^4 x + c$

ⓓ $\frac{1}{3}\tan^3 x + c$

$\int$ ظا^2 س ء س = ...

(أ) ظا س - س + ث

(ب) ظا س + س + ث

(جـ) قا^4 س + ث

(د) $\frac{1}{3}$ ظا^3 س + ث

$$\int \tan^2 x \; dx$$

$$= \int (\sec^2 x - 1) \, dx$$

$$= \tan x - x + C$$

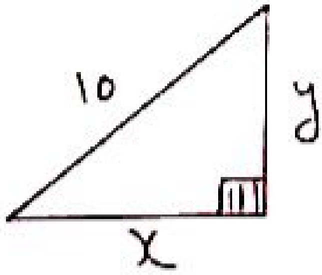
⑭ If the length of the hypotenuse of a right angled triangle equals 10 cm .find the length of the two legs of the right angled triangle when the area of the triangle is as maximum as possible

إذا كان طول وتر مثلث قائم الزاوية ١٠ سم فأوجد طول كل من ضلعيه الآخرين عندما تكون مساحة المثلث أكبر ما يمكن

$$x^2 + y^2 = 100$$

$$y^2 = 100 - x^2$$

$$\boxed{y = (100 - x^2)^{1/2}} \rightarrow (1)$$



$$A = \frac{1}{2} x y$$ from (1)

$$= \frac{1}{2} x (100 - x^2)^{1/2}$$

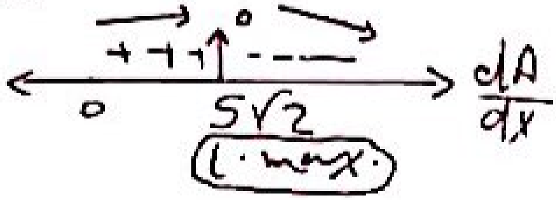
$$\frac{dA}{dx} = \frac{1}{2} \left[x\left(\frac{1}{\cancel{2}}\right)(100 - x^2)^{-1/2}(-2x) + (100 - x^2)^{1/2}(1) \right]$$

$$\frac{dA}{dx} = \frac{1}{2} \left[\frac{-x^2}{\sqrt{100 - x^2}} + \sqrt{100 - x^2} \right]$$

$$0 = \frac{1}{2} \left[\frac{-x^2 + 100 - x^2}{\sqrt{100 - x^2}} \right]$$

$$\therefore -2x^2 + 100 = 0$$

$$x = \boxed{5\sqrt{2}}\ c$$



and $y = (100 - 50)^{1/2}$

$$y = \boxed{5\sqrt{2}}\ c$$

⑮ Find the area of the region bounded by the two curves:

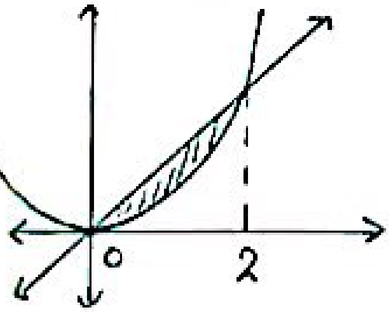
$y = x^2 , y = 2x$.

أوجد مساحة المنطقة المحصورة بين المنحنيين ص = س٢ ، ص = ٢ س

$$x^2 = 2x$$

$$x(x-2) = 0$$

$$\boxed{x = 0} , \boxed{x = 2}$$



$$\text{area} = \int_0^2 (y_1 - y_2)\,dx$$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$= 4 - \frac{8}{3}$$

$$= \frac{4}{3}$$

$$= \boxed{1\frac{1}{3}} \text{ square unit.}$$

⑯

If $\int_{-2}^{3} f(x)dx = 12$,

$\int_{-2}^{5} f(x)dx = 16$,

then $\int_{3}^{5} f(x)dx =$……..

ⓐ -28 ⓑ -4

● 4 ⓓ 28

إذا كان $\int_{-٢}^{٣}$ د(س) ءس = ١٢ ،

$\int_{-٢}^{٥}$ د(س) ءس = ١٦

فإن $\int_{٣}^{٥}$ د(س) ءس = ...

ⓐ ٢٨ ب -٤

جـ ٤ د ٢٨

$$\int_{-2}^{5} f(x)\,dx = \int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$16 = 12 + \int_{3}^{5} f(x)\,dx$$

$$\int_{3}^{5} f(x) = 16-12$$

$$= \boxed{4}$$

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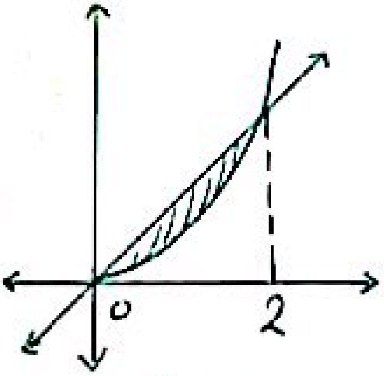
⑰ Find the volume of the solid generated by revolving the region bounded by the two curves: $y = x$, $y = \frac{1}{2}x^2$ a complete revolution about the x-axis .

أوجد حجم الجسم الناشئ من دوران المنطقة المحصورة بين المنحنيين: ص = س، ص = $\frac{1}{٢}$ س٢ حول محور السينات دورة كاملة.



$$y_1 = y_2$$

$$x = \frac{1}{2}x^2$$

$$x\left(\frac{1}{2}x - 1\right) = 0$$

$$\boxed{x = 0} , \boxed{x = 2}$$

$$V = \pi\left[\int_0^2 x^2\,dx - \int_0^2 \left(\frac{1}{2}x^2\right)^2 dx\right]$$

$$= \pi\left(\left[\frac{x^3}{3}\right]_0^2 - \left[\frac{1}{4}\frac{x^5}{5}\right]_0^2\right)$$

$$= \pi\left(\frac{8}{3} - 0 - \frac{1}{20}(32) - 0\right]$$

$$= \boxed{\frac{16}{15}\pi}$$ cubic units

⑱ Answer one of the following items

(a) Find : $\int x(x-2)^4\,dx$

(b) Find : $\int x^3 e^{x^2}\,dx$

أجب عن إحدى الفقرتين الآتيتين :

(أ) أوجد $\int$ س (س − ٢)^{٤} ءس.

(ب) أوجد $\int$ س^{٣} هـ$^{س^{٢}}$ ءس.

Let $x-2=Z$, then $dx=dz$

and $x=z+2$

$$\int x(x-2)^4\,dx$$

$$=\int (z+2)z^4\,dz$$

$$=\int \left(z^5+2z^4\right)dz$$

$$=\frac{z^6}{6}+\frac{2}{5}z^5+c$$

$$=\frac{1}{6}(x-2)^6+\frac{2}{5}(x-2)^5+c$$

$$\int x^3e^{x^2}\,dx=\frac{1}{2}\int x^2\left(2xe^{x^2}\right)dx$$

Let $y=x^2$, $dz=2xe^{x^2}dx$

Then $dy=2xdx$, $z=\int 2xe^{x^2}dx$

$$=e^{x^2}$$

Then $\int x^3e^{x^2}\,dx=\frac{1}{2}[x^2e^{x^2}$

$$-\int 2x\,e^{x^2}dx]$$

$$\frac{1}{2}\left[x^2e^{x^2}-e^{x^2}\right]+c$$

$$\frac{1}{2}e^{x^2}[x^2-1]+c$$

Booklet

2

①

If $f(x) = \sin 2x \cos 2x$, then $f''\left(\frac{\pi}{3}\right)$ equals .....

ⓐ 8 ● $4\sqrt{3}$

ⓒ zero ⓓ $-4\sqrt{3}$

إذا كان: د (س) = جا ٢س جتا ٢س

فإن دً ($\frac{\pi}{٣}$) تساوي ......

(أ) ٨ (ب) ٤ $\sqrt{٣}$

(ج) صفر (د) -٤ $\sqrt{٣}$

$$f(x) = \frac{1}{2}(2 \sin 2x \cos 2x)$$
$$= \frac{1}{2}(\sin 4x)$$
$$f'(x) = \frac{1}{2}(4) \cos 4x$$
$$= 2 \cos 4x$$
$$f''(x) = 2(4)(-\sin 4x)$$
$$= -8 \sin 4x$$
$$f''\left(\frac{\pi}{3}\right) = -8 \sin 4\left(\frac{\pi}{3}\right)$$
$$= -8 \sin 240^\circ$$
$$= -8(-\sin 60^\circ)$$
$$= -8(-\sqrt{3}/2)$$
$$= \boxed{4\sqrt{3}}$$

②

If $y = 2t^3 + 7$, $z = t^2 - 4$, then the rate of change in y with respect to z equals ……

ⓐ 12   ⓑ 6

ⓒ 2t   ● 3t

إذا كان

ص = ٢ن٣ + ٧ ، ع = ن٢ - ٤

فإن معدل تغير ص بالنسبة إلى ع

يساوي......

(أ) ١٢   (ب) ٦

(ج) ٢ن   (د) ٣ن

$$\frac{dy}{dt} = 6t^2 \quad , \quad \frac{dz}{dt} = 2t$$

$$\frac{dy}{dz} = \frac{dy}{dt} \times \frac{dt}{dz}$$

$$= (6t^2) \times \left(\frac{1}{2t}\right)$$

$$= \boxed{3t}$$

③ If $xy = \sin x \cos x$,

prove that :

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy = 0$$

إذا كان س ص = جا س جتا س

أثبت أن

س $\frac{\text{د}^2\text{ص}}{\text{د س}^2}$ + ٢ $\frac{\text{د ص}}{\text{د س}}$ + ٤ س ص = صفر

$$xy = \frac{1}{2}\sin 2x \rightarrow (1)\ (\text{df.w.r.t}\ x)$$

$$x\frac{dy}{dx} + y(1) = \frac{1}{2}(2)\cos 2x$$

$$x\frac{dy}{dx} + y = \cos 2x \quad (\text{df.w.r.t}\ x)$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx}(1) + \frac{dy}{dx} = -2\sin 2x$$

$$x\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right) = -2\sin 2x$$

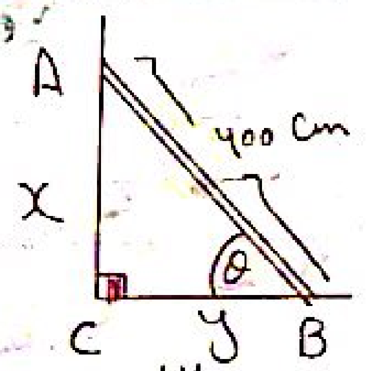
from ① $\sin 2x = 2xy$

$$\therefore x\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right) = -2(2xy)$$

$$x\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right) + 4xy = 0$$

④ A 4-meter ladder is leaning against a vertical wall by its top and on a horizontal ground by its base. If the base of the ladder slides away from the wall at a rate of 20 cm/sec, find the rate of sliding of the top of the ladder when the ladder is inclined to the ground with an angle of measure $\frac{\pi}{3}$.

سلم طوله ٤ أمتار يرتكز بأحد طرفيه على حائط رأسي وبطرفه الآخر على أرض أفقية. فإذا انزلق الطرف الملامس للأرض مبتعداً عن الحائط بمعدل ٢٠سم /ث. احسب معدل هبوط الطرف العلوي للسلم عندما يكون السلم مائلاً على الأرض بزاوية قياسها ($\frac{\pi}{3}$).



$\frac{dy}{dt} = 20$ cm/sec

$\theta = 60°$

$$x^2 + y^2 = 16$$

$$2x\frac{dx}{dt} + 2y\left(\frac{dy}{dt}\right) = 0 \quad \boxed{(-2)}$$

$$x\frac{dx}{dt} + y\left(\frac{dy}{dt}\right) = 0$$

$$x\left(\frac{dx}{dt}\right) + y(20) = 0$$

$$\frac{dx}{dt} = \frac{-20y}{x}$$

$$= \frac{-20\,(400\cos\theta)}{(400\sin\theta)}$$

$$= \frac{-20\cos 60°}{\sin 60°} = \frac{-20}{\sqrt{3}}$$

$$= \boxed{-\frac{20\sqrt{3}}{3}} \text{ cm/sec}$$

⑤

$\lim\limits_{x\to 0}(1+x)^{\frac{1}{5x}}$ equals ....

ⓐ $\frac{1}{5}$ ⓑ e^5

● $e^{\frac{1}{5}}$ ○ $\frac{e}{5}$

نهـا$_{س\to ٠}$ $(١ + س)^{\frac{١}{٥س}}$ تســاوي .....

(أ) $\frac{١}{٥}$ (ب) هـ^{٥}

(جـ) هـ$^{\frac{١}{٥}}$ (د) $\frac{هـ}{٥}$

$$= \lim_{x\to 0}\left[(1+x)^{\frac{1}{x}}\right]^{\frac{1}{5}}$$

$$= \boxed{e^{\frac{1}{5}}}$$

$=$

⑥ If $y = 5 \times 6^x$,

then $\frac{dy}{dx}$ equals ....

- ⓐ 5×6^x
- ● $5 \times 6^x \ln 6$
- ⓒ $5 \times 6^x \log_5 6$
- ⓓ $5 \times 6^x \log_6 5$

إذا كان $ص = ٥ \times ٦^{س}$

فإن $\frac{دص}{دس} =$ ....

- ⓐ ٥×٦^{س}
- ⓑ ٥×٦^{س} لو$_{هـ}$ ٦
- ⓒ ٥×٦^{س} لو$_{٥}$ ٦
- ⓓ ٥×٦^{س} لو$_{٦}$ ٥

$$y = 5 \times 6^x$$

$$\frac{dy}{dx} = \boxed{5 \times 6^x \times \ln 6}$$

⑦ $\int \tan x \, dx$ equals ....

● $\ln|\sec x| + c$

ⓑ $\ln|\cos x| + c$

ⓒ $-\ln|\sec x| + c$

ⓓ $\ln \frac{\sin x}{\cos x}$

∫ظاس ء س تساوي..........

(أ) لوهـ | قاس | + ث

(ب) لوهـ | جتاس | + ث

(جـ) -لوهـ | قاس | + ث

(د) لوهـ $\frac{\text{جاس}}{\text{جتاس}}$

$$\int \tan x \, dx$$

$$= \int \frac{\sin x}{\cos x} dx$$

$$= -\int \frac{-\sin x}{\cos x} dx$$

$$= -\ln|\cos x| + C$$

$$= \ln|\cos x|^{-1} + C$$

$$= \boxed{\ln|\sec x| + C}$$

⑧ If the slope of the tangent to the curve of the function f at any point on it (x, y) is inversely proportion with x and the slope of the tangent equals 2 at $x = 4$ and $y = 2$. Find y in term of x.

إذا كان ميل المماس عند أي نقطة (س ، ص) على منحنى الدالة د يتناسب عكسياً مع س وكان ميل المماس يساوي ٢ عند س = ٤ ، ص = ٢ أوجد ص بدلالة س.

$$\frac{dy}{dx} \propto \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{a}{x}$$

$$\because \frac{dy}{dx} = 2 \text{ at } x = 4$$

$$\therefore 2 = \frac{a}{4} \Rightarrow a = 8$$

$$\therefore \frac{dy}{dx} = \frac{8}{x}$$

$$\therefore y = \int \frac{8}{x} dx$$

$$= 8 \ln|x| + c$$

$$\because y = 2 \text{ at } x = 4$$

$$2 = 8 \ln|4| + c$$

$$\therefore c = 2 - 8 \ln 4$$

$$\therefore y = 8 \ln|x| + 2 - 8 \ln 4$$

$$\therefore y = 8 \ln \frac{|x|}{4} + 2 \quad \#$$

⑨

The curve of the function f:
$f(x) = x^3 - 6x^2$ is convex downwards in the interval ……….

ⓐ]0,2[ⓑ]-1,3[

ⓒ]$-\infty$, 2[●]2,∞[

منحنى الدالة د حيث
د(س) = س³ – ٦ س²
يكون محدباً لأسفل في الفترة ....

(أ)]٠، ٢[(ب)]-١، ٣[

(ج)]-∞، ٢[(د)]٢، ∞[

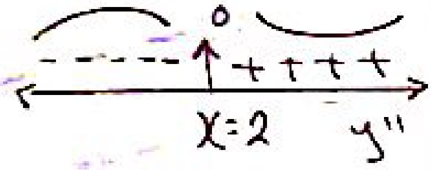
$$f(x) = x^3 - 6x^2$$

$$f'(x) = 3x^2 - 12x$$

$$f''(x) = 6x - 12$$

$$6x - 12 = 0$$

$$\boxed{x = 2}$$



$\therefore$ f(x) is convex downwards on $\boxed{]2,\infty[}$

⑩ If $f(2)$ is an absolute minimum value for the function $f : f(x) = x^2 - kx + 5$ on the interval [-1,5] ,then k = ……

● 4 ⓑ -4

ⓒ 2 ⓓ -2

إذا كانت د (٢) قيمة صغرى مطلقة للدالة د(س) = س٢ – ك س + ٥ على الفترة [-١ ، ٥] فإن ك = ...

(أ) ٤ (ب) -٤

(ج) ٢ (د) -٢

$f(x) = x^2 - kx + 5$

$f'(x) = 2x - k$

$\because$ at $x = 2$ there is an absolute min. , $x \in [-1,5]$

$\therefore f'(2) = 0$

$2(2) - k = 0$

$\boxed{k = 4}$

⑪ The function

$f : f(x) = x|x|$ is .....

● Increasing on R

ⓑ Increasing on $]0,\infty[$ and decreasing on $]-\infty,0[$

ⓒ decreasing on R

ⓓ Increasing on $]-\infty,0[$ and decreasing on $]0,\infty[$

الدالة د حيث

د(س) = س | س | تكون ....

Ⓐ متزايدة على ح

Ⓑ متزايدة على]٠، ∞[ومتناقصة على]-∞، ٠[

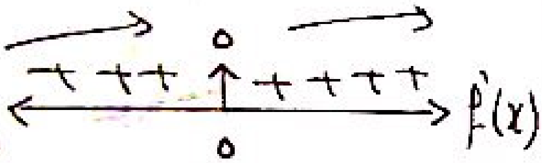
Ⓒ متناقصة على ح

Ⓓ متزايدة على]-∞، ٠[ومتناقصة على]٠، ∞[

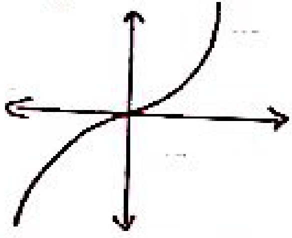
$$f(x) = \begin{cases} x^2 , & x \geq 0 \\ -x^2 , & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x , & x > 0 \\ -2x , & x < 0 \end{cases}$$

differentiable at $x = 0$ because $f'(\bar{0}) = f'(\bar{0}) = 0$



f(x) is increasing on R



⑫ **Answer one of the following items**

(a) Determine the intervals of convexity upwards and downwards for the function f ,then determine the inflection points (if exists) such that :

$f(x) = 6x^3 - x^4$

(b) Find the absolute maximum value of the function f such that:

$f(x) = xe^{-x}, x \in [0,2]$

أجب عن إحدى الفقرتين الآتيتين،

(أ) ابحث فترات تحدب الدالة د ثم أوجد إحداثيات نقط الانقلاب (إن وجدت) حيث

د(س) = ٦س^{٣} - س^{٤}.

(ب) أوجد القيم القصوى المطلقة للدالة د حيث

د(س) = س هـ$^{-س}$ ، س ∈ [٠، ٢]

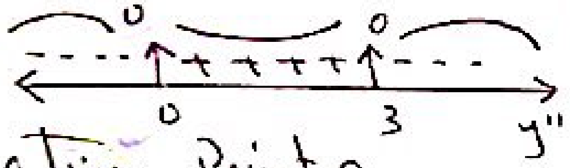
a) $f(x) = 6x^3 - x^4$

$f'(x) = 18x^2 - 4x^3$

$f''(x) = 36x - 12x^2$

$-12x(x-3) = 0$

$\boxed{x=0}$, $\boxed{x=3}$



∴ (0,0) , (3,81) are inflection points

and ~~the~~ Curve is
Convex up. on $R-[0,3]$
, Convex down on $]0,3[$

b) $f(x) = xe^{-x}$

$f'(x) = xe^{-x}(-1) + e^{-x}(1)$

$= -xe^{-x} + e^{-x}$

$= e^{-x}(-x+1)$

$f'(x) = 0$

$\Rightarrow -x+1=0$

$\Rightarrow \boxed{x=1} \in [0,2]$

x	0	1	2
y	0	e^{-1}	$2e^{-2}$

0.37 0.27

The absolute max $= e^{-1} = \frac{1}{e}$ at $\boxed{x=1}$

and the absolute min. = ⓪ at $\boxed{x=0}$

⑬ $\int (4 - \csc x \cot x)dx = \ldots\ldots\ldots$

ⓐ $4x - \csc x + c$

● $4x + \csc x + c$

ⓒ $4x - \cot x + c$

ⓓ $4x + \cot x + c$

∫ (٤ - قتاس ظتاس) ء س = ...

أ ٤س - قتاس + ث

ب ٤س + قتاس + ث

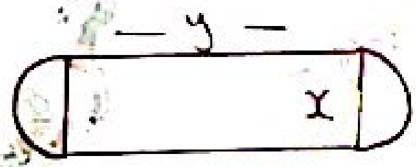
ج ٤س - ظتاس + ث

د ٤س + ظتاس + ث

$= 4X + \csc X + C$

⑭ A playground is in the form of a rectangle ending with semi-circles, if the perimeter of the playground is 420 m, find the maximum area of the playground.

ملعب على شكل مستطيل ينتهي بنصفي دائرتين فإذا كان محيط الملعب ٤٢٠ مترا فأوجد أكبر مساحة له .



$Per. = 2\pi(\frac{x}{2}) + 2y$

$420 = \pi x + 2y$

$\therefore \boxed{y = 210 - \frac{1}{2}\pi x} \rightarrow (1)$

$A = \pi(\frac{x}{2})^2 + xy$ from (1)

$= \frac{1}{4}\pi x^2 + x(210 - \frac{1}{2}\pi x)$

$= \underline{\frac{1}{4}\pi x^2} + 210x - \underline{\frac{1}{2}\pi x^2}$

$= -\frac{1}{4}\pi x^2 + 210x$

$\frac{dA}{dx} = -\frac{1}{2}\pi x + 210$

$0 = -\frac{1}{2}\pi x + 210$

$\pi x = 420 \Rightarrow \boxed{x = \frac{420}{\pi}}$

↓

$$\therefore \; y = 210 - \frac{1}{2}\cancel{\pi}\left(\frac{420}{\cancel{\pi}}\right)$$

$$y = 0$$

$$\because \; \frac{d^2A}{dx^2} = -\frac{1}{2}\pi \quad \therefore \text{l. max}$$

$$\therefore \; A = -\frac{1}{4}\pi\left(\frac{420}{\pi}\right)^2 + 210\left(\frac{420}{\pi}\right)$$

$$= \frac{1}{4}\cancel{\pi}\left(\frac{(420)^2}{\pi^2}\right) + 210\left(\frac{420}{\pi}\right)$$

$$= \frac{-44100}{\pi} + \frac{88200}{\pi}$$

$$= \boxed{\frac{44100}{\pi}} \; m^2$$

$$\simeq \boxed{14037.5} \; m2$$

⑮

If $\int_2^5 f(x)dx = 4$,

then $\int_2^5 (3f(x) - 1)dx = \cdots\cdots$

● 9　　ⓑ 11

ⓒ 12　　ⓓ -8

إذا كان $\int_{٢}^{٥}$ د(س) ءس = ٤

فإن $\int_{٢}^{٥}$ | ٣ د(س) - ١| ءس = ...

(أ) ٩　　(ب) ١١

(جـ) ١٢　　(د) -٨

$$\int_2^5 (3f(x)-1)\,dx$$

$$= 3\int_2^5 f(x) - \int_2^5 1\,dx$$

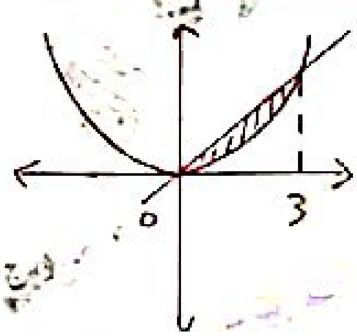
$$= 3(4) - [x]_2^5$$

$$= 12 - (5-2)$$

$$= \boxed{9}$$

⑯ Find the area of the region bounded by the two curves: $y = x^2$, $y = 3x$.

أوجد مساحة المنطقة المحصورة بين المنحنيين ص = س٢ ، ص = ٣ س



$y_1 = y_2$

$x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\boxed{x=0}$, $\boxed{x=3}$

at $x = 1 \in [0,3]$

$\therefore \quad y_2 > y_1$

$\therefore \quad A = \int_0^3 (3x - x^2)\,dx$

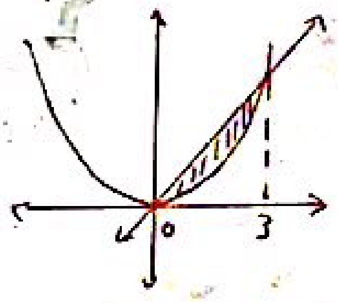
$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$

$= \frac{3}{2}(9) - \frac{1}{3}(27) - 0$

$= \boxed{4.5}$ sq. unit

⑰ Find the volume of the solid generated by revolving the region bounded by the two curves :
$y = x$, $y = \frac{1}{3}x^2$ a complete revolution about the x-axis .

أوجـد حجـم الجسم الناشئ من دوران المنطقة المحصورة بين المنحنيين ص = س ، ص = $\frac{١}{٣}$ س^{٢} حول محور السينات دورة كاملة.



$y_1 = y_2$

$x = \frac{1}{3}x^2$

$3x = x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\boxed{x=0}$, $\boxed{x=3}$

$\boxed{y_1 > y_2}$

$$V = \pi \int_0^3 \left(x^2 - \frac{1}{9}x^4\right) dx$$

$$= \pi \left[\frac{x^3}{3} - \frac{1}{45}x^5\right]_0^3$$

$$= \pi \left[\frac{27}{3} - \frac{1}{45}(3)^5\right] - 0$$

$$= \boxed{\frac{18}{5}\pi} \text{ Cubic units}$$

⑱ **Answer one of the following items**

(a) Find : $\int x\sqrt{x+3}\,dx$

(b) Find $\int 6x\,e^{2x}\,dx$

أجب عن إحدى الفقرتين الآتيتين:

(أ) أوجد ∫ س √س + ٣ ء س

(ب) أوجد ∫ ٦ س هـ^٢س ء س

a) $\int x(x+3)^{1/2}\,dx$

let $x+3 = z \Rightarrow \boxed{x = z-3}$

$\therefore\ dx = dz$

$= \int (z-3)(z)^{1/2}\,dz$

$= \int (z^{3/2} - 3z^{1/2})\,dz$

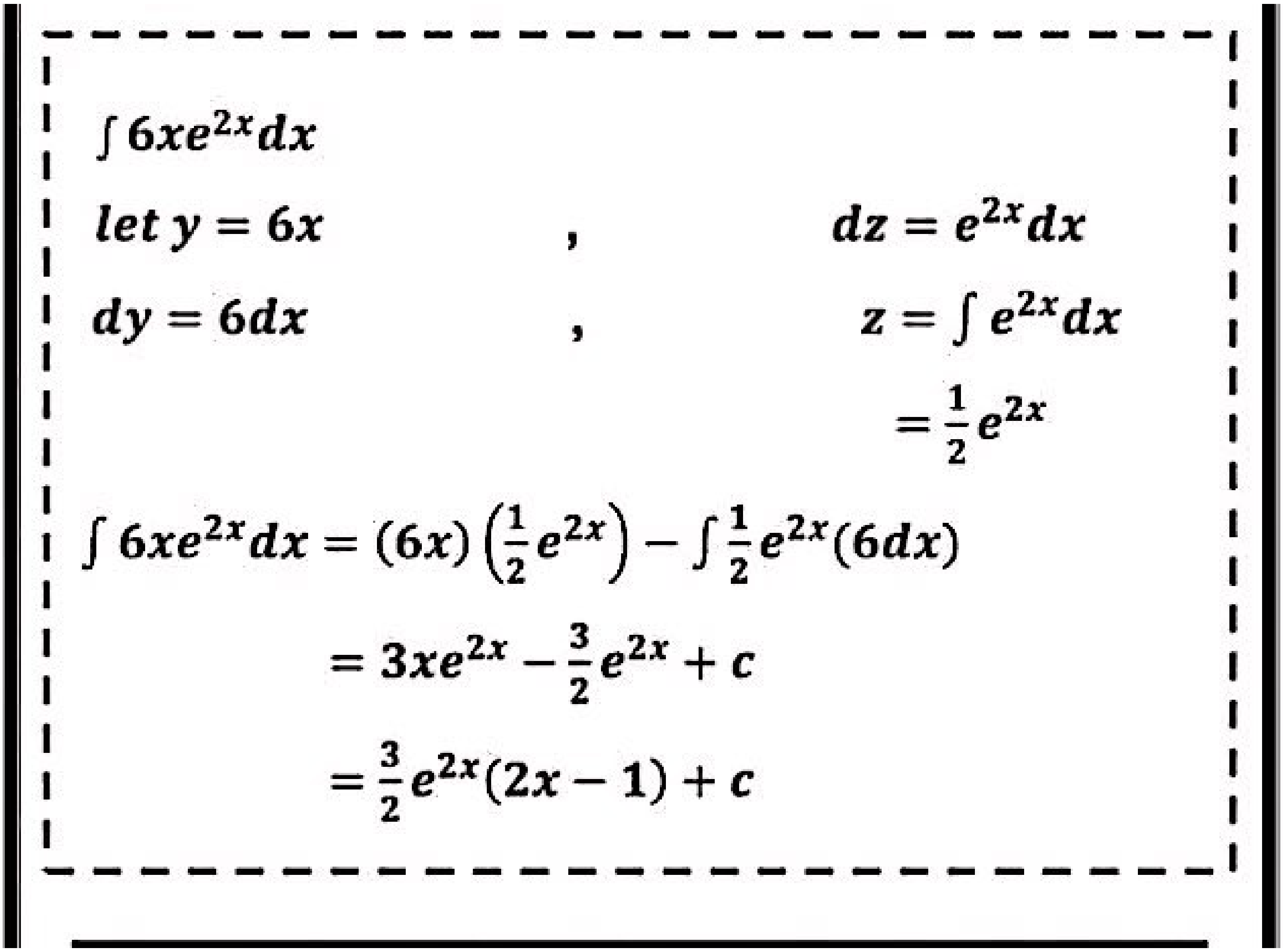
$= \dfrac{z^{5/2}}{5/2} - \dfrac{3z^{3/2}}{3/2} + C$

$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$

$= \frac{2}{5}(x+3)^{3/2}\left[(x+3) - 5\right] + C$

$= \frac{2}{5}\sqrt{(x+3)^3}\,(x-2) + C$

$= \frac{2}{5}(x+3)\sqrt{x+3}\,(x-2) + C$



$\int 6xe^{2x}dx$

$let\ y = 6x$, $dz = e^{2x}dx$

$dy = 6dx$, $z = \int e^{2x}dx$

$= \frac{1}{2}e^{2x}$

$$\int 6xe^{2x}dx = (6x)\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}(6dx)$$

$$= 3xe^{2x} - \frac{3}{2}e^{2x} + c$$

$$= \frac{3}{2}e^{2x}(2x-1) + c$$

Booklet

3

(1)If $x^2 + y^2 = 2xy$ then y''=......

(a) zero (b) 1 (c) 2 (d)-1

(2) $\int_{-2}^{2}|x|dx$ =.......

(a) -4 (b) zero (c) 2 (d) 4

(3) Find the equation of the tangent to the curve: $x = tan\,\theta$, $y = sec\,\theta$ at $\theta = \frac{\pi}{4}$

(4)If $y = \frac{z+1}{z-1}$, $x = \frac{z-1}{z+1}$, prove that : $x \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right) = 0$

(5) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$ =.....

(a) e (b) e^2 (c) e^3 (d) e^6

(6) If $f(x)$=e^{2x} ,then f'' (x)= ....

(a) $f(x)$ (b) $2f(x)$ (c) $3f(x)$ (d) $4f(x)$

(7) Find the value of: $\int_0^{\frac{\pi}{2}} x \sin x\, dx$

(8) Answer one of the following items:

a) Find the interval(s) in which the curve of the function $f(x) = \sin 2x$ is decreasing, increasing, convex upward, convex downward also determine the inflection point and the absolute maximum and minimum values of the function in the interval $[0, 2\pi]$

b) Find the equation of the curve of this function $y = f(x)$. In which the point $(0, 0)$ is an inflection point, $\frac{d^2y}{dx^2} = ax + b$ and $(-2, 16)$ is a local maximum point.

(9) If $\int_1^k \frac{dx}{x} = 1$, then $k =$.....

(a) e (b) 10 (c) ln 10 (d) $\log e$

(10) $\int_{-\pi}^{\pi} \tan^3 x\, dx =$.......

(a) zero (b) π (c) $-\pi$ (d) 2π

(11) If the gas leaks from a spherical balloon at a constant rate and the length of its diameter decreased from 22 cm to 12 cm in 20 second ,find the rate of change of the its volume when the length of the radius equals 7 cm.

(12)Find a point (x , y) on the curve $x^2+y^2=100$ in which the distance between it and the point (15 , 20) is as small as possible

(13)If $f(x) = log\,x$ then $f\,'(x)$ =……

(a) 1 (b) $\frac{1}{x}$ (c) $\frac{\log e}{x}$ (d) $\frac{1}{x \log e}$

---

(14) The volume of the solid generated by the revolving of the region bounded by the straight line $y = 2x$ and the straight line $x = 3$ a complete revolution about the x-axis is as the volume of …………

(a) a sphere whose diameter length is 3 unit

(b) a sphere whose diameter length is 6 unit

(c) a cone of height equals the radius of its base = 3 units

(d) a right cylinder of height equals the radius of its base = 3 units

---

(15) Answer one of the following items:

(a)Find the area of the region bounded by the curve $y = 9 - x^2$ and the x-axis

(b) Find the volume of the solid generated by the revolution of the region bounded by two curves: $y = x^2, y = \sqrt{x}$ a complete revolution about the y-axis

---

(16) Find $\frac{dy}{dx}$ If $y = x\,ln\,x - x$,then deduce the value of $\int_1^e \ln x^2\, dx$

---

(17) $\lim\limits_{x \to 0} \frac{1-e^{-x}}{x}$ =……..

(a) -1 (b)0 (c) 1 (d) e

---

(18) $\int \frac{e^x}{1+e^x}\,dx =$ ……

(a) $\ln(1 + e^x) + c$ (b) $\log(1 + e^x) + c$

(c) $\ln \frac{1}{1+e^x} + c$ (d) $\log \frac{1}{1+e^x} + c$

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Model answer

1] a]

$$x^2+y^2=2xy$$

$$2x+2yy'=2[xy'+y] \quad \div 2$$

$$x+yy'=xy'+y$$

$$yy'-xy'=y-x$$

$$y'(y-x)=y-x$$

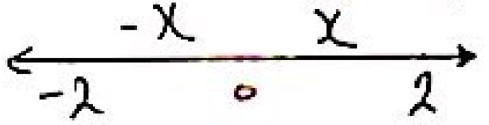
$$y'=\frac{y-x}{y-x}$$

$$y'=1$$

$$\boxed{y''=0}$$

2] a]

$$|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$$



$$\int_{-2}^{2}|x|\,dx$$

$$=\int_{-2}^{0}-x\,dx+\int_{0}^{2}x\,dx$$

$$=\left[-\frac{x^2}{2}\right]_{-2}^{0}+\left[\frac{x^2}{2}\right]_{0}^{2}$$

$$=0-\left(-\frac{4}{2}\right)+\left[\frac{4}{2}\right]$$

$$=2+2=\boxed{4}$$

3] $x=\tan\theta$, $y=\sec\theta$

$$\frac{dx}{d\theta}=\sec^2\theta \qquad \frac{dy}{d\theta}=\sec\theta\tan\theta$$

$$\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$$

$$=\cancel{\sec\theta}\tan\theta\times\frac{1}{\sec^{\cancel{2}}\theta}$$

$$=\frac{\tan\theta}{\sec\theta}$$

$$=\frac{\sin\theta}{\cancel{\cos\theta}}\times\frac{\cancel{\cos\theta}}{1}$$

$$=\sin\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}}=\sin\frac{\pi}{4}$$

$$=\boxed{\frac{1}{\sqrt{2}}}$$

$x=\tan\frac{\pi}{4}$, $y=\sec\frac{\pi}{4}$

$=①$, $=\sqrt{2}$

Point $(1,\sqrt{2})$

$$\frac{y-\sqrt{2}}{x-1}=\frac{1}{\sqrt{2}}$$

$$y\sqrt{2}-2=x-1$$

$$\boxed{x-\sqrt{2}y+1=0}$$

4] $xy=1$

$$x\frac{dy}{dx}+y=0$$

$$x\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)(1)+\frac{dy}{dx}=0$$

↓

↓

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$$ #

5 d

let $\frac{2}{x}=y$

$x\to\infty \Rightarrow \boxed{y\to 0}$

$x=\frac{2}{y}$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$$

$$=\lim_{y\to 0}(1+y)^{3\left(\frac{2}{y}\right)}$$

$$=\lim_{y\to 0}\left[(1+y)^{\frac{1}{y}}\right]^{6}$$

$$=\boxed{e^6}$$

6 d

$f(x)=e^{2x}$

$f'(x)=2e^{2x}$

$f''(x)=4e^{2x}$

$=4f(x)$

7 let $y=x$, $dz=\sin x\,dx$

$dy=dx$, $z=\int \sin x\,dx$

$=-\cos x$

$$\int_0^{\frac{\pi}{2}} x\sin x\,dx$$

$$=-x\cos x-\int -\cos x\,dx$$

$$=\left[-x\cos x+\sin x\right]_0^{\pi/2}$$

8 a)

$f(x)=\sin 2x$

Corr. $[0,\pi]$

$f'(x)=2\cos 2x$

$f''(x)=-4\sin 2x$

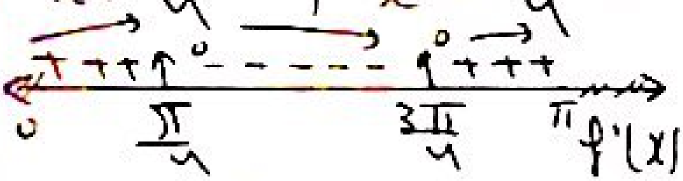
$f'(x)=0$

$2\cos 2x=0$

$\cos 2x=0$

$2x=\frac{\pi}{2}$, $2x=\frac{3\pi}{2}$

$x=\frac{\pi}{4}$, $x=\frac{3\pi}{4}$



incr. on $]0,\frac{\pi}{4}[\,\cup\,]\frac{3\pi}{4},\pi[$

decr. on $]\frac{\pi}{4},\frac{3\pi}{4}[$

$(\frac{\pi}{4},1)$ L. max point

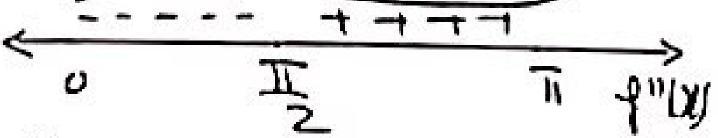
$(\frac{3\pi}{4},-1)$ L. min Point

$f''(x)=0$

$-4\sin 2x=0$

$2x=0$, $2x=180$

$\boxed{x=0}$, $\boxed{x=90}$



Convex up on $]0,\frac{\pi}{2}[$

Convex down on $]\frac{\pi}{2},\pi[$

$(\frac{\pi}{2},0)$ inflection Point

↓

b) $\frac{d^2y}{dx^2} = ax + b$

(0,0) inflection Point

(-2,16) L. max

$\frac{d^2y}{dx^2} = 0$ at $(0,0)$

$a(0) + b = 0 \Rightarrow \boxed{b = 0}$

$\frac{d^2y}{dx^2} = ax$

$\frac{dy}{dx} = \int ax\,dx$

$= \frac{1}{2}ax^2 + C$

$\frac{dy}{dx} = 0$ at $(-2,16)$

$\frac{1}{2}a(-2)^2 + C = 0$

$\boxed{2a + C = 0} \to (1)$

$y = \int(\frac{1}{2}ax^2 + c)dx$

$y = \frac{1}{6}ax^3 + Cx + d$

$(0,0) \in$ Curve

$\boxed{d = 0}$

$y = \frac{1}{6}ax^3 + Cx$

$(-2,16) \in$ Curve

$16 = \frac{1}{6}(-2)^3 a + C(-2)$

$16 = \frac{-4}{3}a - 2C$

$48 = -4a - 6C$ (÷2)

$\boxed{24 = -2a - 3C} \to (2)$

fr ①, ②

$\boxed{C = -12}$

$\boxed{a = 6}$

$y = \frac{1}{6}(6)x^3 - 12x$

$\boxed{y = x^3 - 12x}$ #

9 a

$\int_1^k \frac{dx}{x} = 1$

$[\ln|x|]_1^k = 1$

$\ln|k| - \ln 1 = 1$

$\ln|x| - 0 = 1$

$\ln|x| = 1$

$|x| = e$

$\boxed{x = e}$

10 a

Correction $\int_{-\pi}^{\pi} \sin^3 x\,dx$

because tan x (undef. at $\frac{\pi}{2}$)

↓

↓

$$f(-x) = \sin^3(-x)$$
$$= -\sin^3 x$$
$$= -f(x)$$

$\therefore \sin^3 x$ (odd. fun.) and Cont. on $[-\pi, \pi]$

$$\therefore \int_{-\pi}^{\pi} \sin^3 x \, dx = \boxed{\text{Zero}}$$

11

r changes from 11 → 6 cm in 20 sec.

$$\boxed{V = \frac{4}{3}\pi r^3}$$

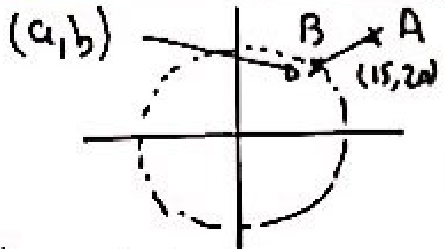
$$\frac{dr}{dt} = \frac{6-11}{20} = \frac{-1}{4}$$

$$\frac{dv}{dt} = \frac{4}{3}\pi(3r^2)\left(\frac{dr}{dt}\right)$$
$$= 4\pi(r^2)\left(\frac{dr}{dt}\right)$$

$\underline{r = 7}$ cm when $\frac{dr}{dt} = \frac{-1}{4}$

$$\therefore \frac{dv}{dt} = 4\pi(7)^2\left(\frac{-1}{4}\right)$$
$$= \boxed{-49\pi} \text{ cm}^3/\text{sec}$$

12



let $(a,b) \in$ curve ↗

$$\therefore a^2 + b^2 = 100$$
$$\boxed{b^2 = 100 - a^2} \rightarrow (1)$$

$$AB = \sqrt{(a-15)^2 + (b-20)^2}$$
$$S = \sqrt{(a-15)^2 + (b-20)^2}$$
$$S^2 = (a-15)^2 + (b-20)^2$$
$$S^2 = a^2 - 30a + 225 + \underline{b^2} - 40b + 400$$
$$S^2 = \cancel{a^2} - 30a + 625 + \underline{\underline{100 - a^2}} - 40\sqrt{100 - a^2}$$

$$\boxed{b = \sqrt{100 - a^2} \text{ (because } (15,20) \in \text{1st quad)}}$$

$$\therefore S^2 = -30a - 40(100 - a^2)^{\frac{1}{2}} + 725$$

(der. w.r.t x)

$$2S\frac{dS}{da} = -30 - 20(100 - a^2)^{-\frac{1}{2}} \times (-2a) + 0$$

$$0 = -30 + \frac{40a}{\sqrt{100 - a^2}}$$

$$3\cancel{0} = \frac{4\cancel{0}a}{\sqrt{100 - a^2}}$$

$$3\sqrt{100 - a^2} = 4a \quad \text{sq.}$$

$$9(100 - a^2) = 16a^2$$ ↓

↓

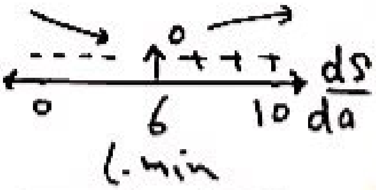
$$900 - 9a^2 = 16a^2$$

$$25a^2 = 900$$

$$a^2 = 36$$

$$a = \pm 6$$ (-ve ref.) 1st quad.

∴ $a = 6$



$$b = \sqrt{100-36} = \boxed{8}$$

∴ at Point (6,8) The dist. is min.

13 **c**

$$f(x) = \log x$$

$$f'(x) = \frac{1}{x \ln 10}$$

$$= \frac{1}{x\left(\frac{\log 10}{\log e}\right)}$$

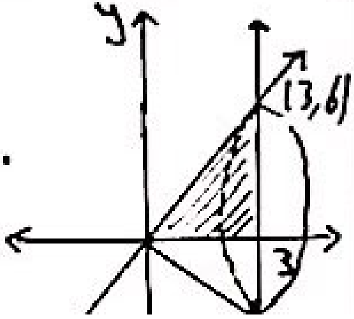
$$= \frac{1}{x\left(\frac{1}{\log e}\right)}$$

$$= \frac{\log e}{x}$$

14 **b**

$$y = 2(3)$$
$$= 6$$

↓



$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (6)^2 (3)$$

$$= 36\pi$$

let $V_{Cone} = V_{Sphere}$

$$36\cancel{\pi} = \frac{4}{3}\cancel{\pi}(r)^3$$

$$r^3 = 27$$

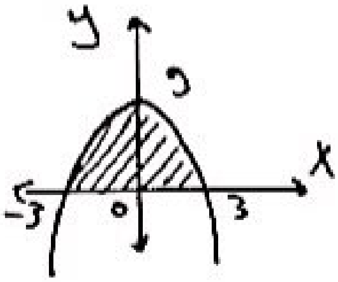
$$\boxed{r = 3}$$

∴ $V_{Cone} = V_{Sphere}$ of radius 3 cm

15 a)

$$9 - x^2 = 0$$

$$\boxed{x = \pm 3}$$



$$\text{area} = \int_{-3}^{3}(9-x^2)\,dx$$

$$= 2\int_{0}^{3}(9-x^2)\,dx$$

$$= 2\left[9x - \frac{x^3}{3}\right]_0^3$$

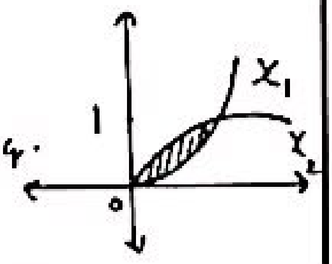
$$= 2\left[27 - 9\right]$$

$$= \boxed{36}$$ sq. unit

b) $\boxed{y_1 = x^2}$

$y = \sqrt{x}$ Sq.

$\boxed{y_2^2 = x}$



$$x^2 = \sqrt{x} \Rightarrow x^4 = x$$

$$\Rightarrow x(x^3 - 1) = 0$$

$\boxed{x = 0}$, $\boxed{x = 1}$

$\boxed{y = 0}$ $\boxed{y = 1}$

$$x_1 > x_2$$

$$V = \pi \int_0^1 (x_1^2 - x_2^2)\,dy$$

$$= \pi \int_0^1 (y - y^4)\,dy$$

$$= \pi \left[\frac{1}{2}y^2 - \frac{1}{5}y^5\right]_0^1$$

$$= \pi \left[\frac{1}{2} - \frac{1}{5}\right]$$

$= \boxed{\frac{3}{10}\pi}$ Cubic unit

$\boxed{16}$ $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x - 1$

$= 1 + \ln x - 1$

$= \boxed{\ln x} \to$ ①

$$\int_1^e \ln x^2\,dx$$

$$= \int_1^e 2\ln x\,dx$$

from ①

$$\ln x = \frac{dy}{dx}$$

$$\therefore \int_1^e \ln x^2 = 2\int_1^e \frac{dy}{dx} \cdot dx$$

$$= 2\int_{-1}^{0} dy$$

$x = 1 \Rightarrow y = 1\ln(1) - 1$

$= \boxed{-1}$

$x = e \Rightarrow y = e\ln e - e$

$= e(1) - e$

$= 0$

$$\therefore \int_1^e \ln x^2 = 2[y]_{-1}^0$$

$= 2(0) - 2(-1)$

$= \boxed{2}$

$\boxed{17}$ $\boxed{c}$

$$\lim_{x \to 0} \frac{1 - e^{-x}}{x}$$

$$= \lim_{(-x) \to 0} \frac{e^{(-x)} - 1}{(-x)} = \lim_{y \to 0} \frac{e^y - 1}{y}$$

$= \ln e = \boxed{1}$

$\boxed{18}$ $\boxed{c}$

$$= \int \frac{e^x}{1 + e^x}\,dx$$

$$= \ln(1 + e^x) + C$$