Booklet (3) Exams

Algebra and Solid Geometry

Third Sec

منتدى توجيه الرياضيات

أ/ عادل إدوار

Arab Republic of Egypt

Ministry Of Education

Subject: Algebra & Analytic Geometry

Time Allowed: Two Hours

Experimental Test for third secondary stage 2017/2018

Answer the following questions:-

(1) If ${}^{n}C_{r} : {}^{n}P_{r} = \frac{1}{6}$ then $\underline{|r-3}$ = ...

(a) zero (b) 1 (c) 3 (d)6

....
....
....
....
....
....
....
....
....
....
....
....
....

(2) The last term in the expansion of $(2-x)^5\,(2+x)^5$ according to the ascending power of x is ....

(a) x^5 (b) $-x^5$ (c) x^{10} (d) $-x^{10}$

....
....
....
....
....
....
....
....
....
....
....

(3) The length of the perpendicular drawn from point (1 , -3 , 4) to the x- axis equals ....length unit.

(a) 1 (b) 3 (c) 4 (d) 5

(4) The measure of the acute angle between the two vectors (0 , - b , b) , (b , − b , 0) equals ....° where b is a constant $\neq 0$

(a)30 (b) 45 (c)60 (d) 90

(5) Answer one of the following items:

In the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ according to the descending power of x .

(a) Find the ratio between the term free of x and the coefficient of the eighth term.

(b) Find the ratio between the coefficient of the middle term and the coefficient of the 10th term

....
....
....
....
....
....
....
....
....
....
....
....
....

(6) Find the principle amplitude of the complex number $z = -1 - i$

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{5\pi}{4}$

....
....
....
....
....
....
....
....
....
....
....

(7) The value of: $\left(\frac{1}{1+\omega i}-\frac{\omega+i}{1+\omega^2 i}\right)^8 = \dots\dots\dots$

(a) 16 (b) 8 (c) $16i$ (d) $8i$

(8) Without expanding the determinant,

Solve the following equation : $\begin{vmatrix} 3x & 2x & x \\ x & 2x & 3x \\ x & -x & 0 \end{vmatrix} = 96$

(9) The number of the solutions of the system: $3x + 2y = 4$, $2x + 3y = 6$ is.......

(a) Zero (b) 2 (c) 1 (d) infinite number of solutions

(10) If the middle term of the expansion of: $\left(\frac{x}{3} + \frac{3}{x}\right)^{2n}$ is the seventh term, then $n =$ ....

(a) 6 (b) 7 (c) 12 (d) 14

(11) The volume of the parallelepiped in which three adjacent sides of it are represented by the vectors $\vec{A}=(0,0,2)$, $\vec{B}=(8,0,0)$, $\vec{C}=(0,4,0)$ =..........cubic unit

(a) 8 (b) 16 (c) 32 (d) 64

....
....
....
....
....
....
....
....
....
....
....
....
....

(12) Answer one of the following items:

a) Find the different forms of the equation of the straight line passing through the point (4 , -2 , 5) and the vector (1 , -2 , 2) is a direction vector of it.

b) If $\vec{C}=(2,1,-2)$ and $\vec{C}+\vec{B}=\vec{C}\times\vec{B}$, then find $\|\vec{B}\|$

....
....
....
....
....
....
....
....
....
....
....
....
....
....

....
....
....
....
....
....
....
....
....
....

(13) The diameter length of the sphere $x^2 + y^2 + z^2 \ -6x + 4y - 2z = 11$ equals ....length unit

(a) 5 (b) 10 (c) 11 (d) 15

....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....

(14) Solve the following systems of equations using the inverse matrix:

$$2x + z = 5 \,, x + y = 3 \,, z - y = 1$$

(15)If the two straight lines $\frac{x+1}{3}=\frac{2-y}{3}=\frac{z-3}{6}$, $\frac{x}{2}=\frac{y+1}{-2}=\frac{z-1}{k}$ are parallel, then k =......

(a)-6 (b)4 (c)6 (d)-4

(16) If the straight line $\frac{x}{3}=y=\frac{z}{-3}$ is parallel to the plane $x+ky+2z+k=0$, then k =......

(a)-3 (b)1 (c)3 (d)6

(17) If $Z_1= 1+\sqrt{3}\, i$, $Z_2=\sqrt{2}\,(cos45^0 + i \sin 45^0)$, Find the exponential form of : $(z_1 \div z_2)^6$

....

(18) The length of the perpendicular drawn from the point (1 , -6, 3) to the plane

$\vec{r} \odot (2 , -2 , 1)=5$

....

(19) Find the point of intersection of the straight line : $x = y = z$ and the plane:

$\vec{r} \odot (1, 2, 3) = 12$

Model answer

1 (b)

$$\frac{{}^nC_r}{{}^nP_r} = \frac{1}{6}$$

$$6\,{}^nC_r = {}^nP_r$$

$$6\left[\frac{\cancel{{}^nP_r}}{\lfloor r}\right] = \cancel{{}^nP_r}$$

$$\lfloor r = 6 \rightarrow \boxed{r=3}$$

$$\therefore \lfloor r-3 = \lfloor 3-3 = \lfloor 0 = \boxed{1}$$

2 (d)

$$(2-x)^5(2+x)^5$$

$$= [(2-x)(2+x)]^5$$

$$= [4-x^2]^5$$

last term $= T_6$

$$= {}^5C_5(-x^2)^5(4)^0$$

$$= \boxed{-x^{10}}$$

3 (d)

The length of the $\perp$

$$= \sqrt{y^2+z^2}$$

$$= \sqrt{(-3)^2+(4)^2}$$

$$= \boxed{5}$$

4 (c)

$$\cos\theta = \frac{\vec{A} \odot \vec{B}}{\|\vec{A}\|\,\|\vec{B}\|}$$

$$= \frac{(0,-b,b)\odot(b,-b,0)}{\sqrt{b^2+b^2}\,\sqrt{b^2+b^2}}$$

$$= \frac{0+b^2+0}{\sqrt{2b^2}\,\sqrt{2b^2}}$$

$$= \frac{\cancel{b^2}}{\sqrt{2}\cancel{b}\sqrt{2}\cancel{b}}$$

$$= \frac{1}{2}$$

$$\boxed{\theta = 60}$$

5 a) $\left(x^2+\frac{1}{x}\right)^{12}$

$$T_{r+1} = {}^{12}C_r\left(\frac{1}{x}\right)^r(x^2)^{12-r}$$

$$= {}^{12}C_r(x)^{-r}(x)^{24-2r}$$

$$= {}^{12}C_r(x)^{24-3r}$$

$$24-3r=0 \Rightarrow \boxed{r=8}$$

$$\therefore T_9 = {}^{12}C_8(x)^0 = {}^{12}C_8$$

$$T_8 = {}^{12}C_7\left(\frac{1}{x}\right)^7(x^2)^5$$

Coeff. of $T_8 = {}^{12}C_7$

$\therefore \dfrac{\text{Term free of } x}{\text{Coeff. of } T_8} = \dfrac{{}^{12}C_8}{{}^{12}C_7}$

$= \dfrac{12-8+1}{8}$

$= \dfrac{5}{8}$

b) order $= \dfrac{12+2}{2} = 7$

$T_7 = {}^{12}C_6 \left(\frac{1}{x}\right)^6 (x^2)^6$

Coeff. of $T_7 = {}^{12}C_6$

$T_{10} = {}^{12}C_9 \left(\frac{1}{x}\right)^9 (x^2)^3$

Coeff. of $T_{10} = {}^{12}C_9$

$\dfrac{\text{Coeff. of } T_7}{\text{Coeff. of } T_{10}} = \dfrac{{}^{12}C_6}{{}^{12}C_9}$

$= \dfrac{21}{5}$

$= \boxed{21 : 5}$

6) c

$\theta = -\pi + \tan^{-1}\left(\frac{y}{x}\right)$

$= -180° + 45°$

$= -135$

$= \boxed{\dfrac{-3\pi}{4}}$

7) a

$\left(\dfrac{1}{1+\omega i} - \dfrac{\omega + i}{1+\omega^2 i}\right)^8$

$= \left[\dfrac{1+\omega^2 i - (1+\omega i)(\omega + i)}{(1+\omega i)(1+\omega^2 i)}\right]^8$

$= \left[\dfrac{1+\omega^2 i - (\omega + \omega^2 i + i + \omega i^2)}{1+\omega i + \omega^2 i + \omega^3 i^2}\right]^8$

$= \left[\dfrac{1+\cancel{\omega^2 i} - \cancel{\omega} - \cancel{\omega^2 i} - i + \cancel{\omega}}{\cancel{1} + i(\omega + \omega^2) + \cancel{(-1)}}\right]^8$

$= \left[\dfrac{1 - i}{-i}\right]^8$

$= \left[-\dfrac{i^4}{i} + 1\right]^8$

$= \left[-i^3 + 1\right]^8$

$= \left[(i+1)^2\right]^4$

$= \left[\cancel{-1} + 2i + \cancel{1}\right]^4$

$= (2i)^4 = \boxed{16}$

8)

$\begin{vmatrix} 3x & 2x & x \\ x & 2x & 3x \\ x & -x & 0 \end{vmatrix} = 96$

$x^3 \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & 0 \end{vmatrix} = 96$

$\downarrow$

$$\boxed{(C_2 \times 1) + C_1, (C_3 \times 1) + C_1}$$

$$x^3 \begin{vmatrix} 6 & 2 & 1 \\ 6 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 96$$

$$\boxed{(R_1 \times -1) + R_2}$$

$$x^3 \begin{vmatrix} 6 & 2 & 1 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{vmatrix} = 96$$

$$-x^3 \begin{vmatrix} 6 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 96$$

$$-x^3(-12) = 96$$

$$x^3 = 8 \Rightarrow \boxed{x = 2}$$

9 c

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 5 \neq 0$$

$\therefore Rk(A) = 2$

$\therefore Rk(A) = R(A^*) = 2$

$\therefore$ eq. has one Solution

10 a

$$\frac{2n+2}{2} = 7$$

$$n+1 = 7 \Rightarrow \boxed{n = 6}$$

11 d

$$V = |\vec{A} \cdot \vec{B} \times \vec{C}|$$

$$= \begin{vmatrix} 0 & 0 & 2 \\ 8 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$= -8 \begin{vmatrix} 0 & 2 \\ 4 & 0 \end{vmatrix}$$

$$= -8(-8) = \boxed{64}$$

12 a) $\vec{r} = \vec{A} + t\,\vec{d}$

$$(x, y, z) = (4, -2, 5) + t(1, -2, 2)$$

$x = 4 + t$, $y = -2 - 2t$, $z = 5 + 2t$

$\boxed{x - 4 = t}$, $\boxed{\frac{y+2}{-2} = t}$, $\frac{z-5}{2} = t$

$$\therefore \boxed{x - 4 = \frac{y+2}{-2} = \frac{z-5}{2}}$$

b) let $\vec{B} = (a, b, c)$

$\because \vec{C} + \vec{B} = \vec{C} \times \vec{B}$

$\therefore (2, 1, -2) + (a, b, c)$

$= (2, 1, -2) \times (a, b, c)$

$\therefore (a+2, b+1, c-2)$

$$= \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ a & b & c \end{vmatrix}$$

l

$(a+2,\ b+1,\ c-2) = (c+2b,\ -2c-2a,\ 2b-a)$

$\therefore a+2 = c+2b$

$\boxed{a-c-2b=-2} \rightarrow (1)$

$b+1 = -2c-2a$

$\boxed{b+2c+2a=-1} \rightarrow (2)$

$c-2 = 2b-a$

$\boxed{c-2b+a=2} \rightarrow (3)$

from (1), (2), (3)

$\boxed{a=-2},\ \boxed{b=-1},\ \boxed{c=2}$

$\boxed{\vec{B} = (-2,-1,2)}$

---

13 b

$L = (-3),\ K = (2),\ M = -1$

$r = \sqrt{9+4+1-(-11)}$

$= 5$

diameter $= 5 \times 2$

$= \boxed{10}$ unit lgth

---

14 $A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

$\Delta = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$

$= 2(1) - (1) = 1 \neq 0$

$\therefore$ has one sol.

$C = \begin{pmatrix} \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} \end{pmatrix}$

$= \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 2 \\ -1 & 1 & 2 \end{pmatrix}$

$\text{adj}(A) = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$

$A^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5-3-1 \\ -5+6+1 \\ -5+6+2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$S.S = \{(1,2,3)\}$

15) d)

$d_1 = (3, -3, 6)$

$d_2 = (2, -2, k)$

$\vec{d_1} \parallel \vec{d_2}$

$\frac{3}{2} = \frac{6}{k} \Rightarrow \boxed{k = 4}$

16) c)

$\vec{d} = (3, 1, -3)$

$\vec{n} = (1, k, 2)$

$\vec{d} \odot \vec{n} = 0$

$(3, 1, -3) \odot (1, k, 2) = 0$

$3 + k - 6 = 0$

$\boxed{k = 3}$

17)

$Z_1 = 2(\cos 60^\circ + i \sin 60^\circ)$

$Z_2 = \sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$

$\frac{Z_1}{Z_2} = \sqrt{2}(\cos 15^\circ + i \sin 15^\circ)$

$\left(\frac{Z_1}{Z_2}\right)^6 = 8(\cos 90^\circ + i \sin 90^\circ)$

$= \boxed{8 e^{i\left(\frac{\pi}{2}\right)}}$

18)

$(x, y, z) \odot (2, -2, 1) = 5$

$\boxed{2x - 2y + z = 5}$

$h = \frac{|2(1) - 2(-6) + 3 - 5|}{\sqrt{4 + 4 + 1}}$

$= 4$

19)

$\boxed{x = y = z = t} \rightarrow ①$

$(x, y, z) \odot (1, 2, 3) = 12$

$\boxed{x + 2y + 3z = 12} \rightarrow ②$

from ① in ②

$t + 2t + 3t = 12$

$\boxed{t = 2}$

$\therefore x = y = z = 2$

$\therefore$ point of intersection $= \boxed{(2, 2, 2)}$

Good luck

Mr/ George Adel

01225923424