Final Revision

for Third year Secondary

Rule Dynamics

Dynamics

Rectilinear Motion

1) Displacement

- The displacement $\vec{s}$ of a particle is known as the change of its position.
  Displacement $\vec{s} = \Delta \vec{x}$ where $\Delta \vec{x} = \vec{x} - \vec{x_o}$
- If the position of the body at the beginning of measuring the time is at the origin point, then $\vec{x_o} = \vec{0}$ and $\vec{s} = \vec{x}$

2) Velocity

Velocity $\vec{V} = \frac{d\vec{s}}{dt} = \frac{d\vec{x}}{dt}$

- The average velocity (speed) $= \frac{\text{total distance}}{\text{total time}}$
- The average velocity vector $= \frac{\text{Displacement}}{\text{total time}}$
- If the body at the maximum displacement, then $= 0$.
- If the body moves with maximum velocity or with uniform velocity then $= 0$.
- If the body get back to its initial position, then $s = 0$.

3) Speed

If $\vec{V}(t)$ is the velocity vector of a body moving in a straight line, then the speed is the standard quantity expressing the magnitude of the velocity vector

$$The\ speed = \|\vec{V}\| = \left\|\frac{d\vec{x}}{dt}\right\| = \left\|\frac{d\vec{s}}{dt}\right\|$$

4) Acceleration

- If $\Delta\vec{V}$ expresses the change of the velocity vector during time interval Δt then the average acceleration $\vec{a}_a$ is given by the relation $\vec{a}_a = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V}(t+\Delta t)-\vec{V}(t)}{\Delta t}$
- $\vec{a} = \frac{d\vec{V}}{dt}$
- The acceleration is the rate of change of the velocity vector with respect to the time *(the slope of the tangent to the velocity – time graph)*
- $a = V \cdot \frac{dv}{dx}$ It is another form for the acceleration which can be used when the velocity vector $\vec{V}$ is a function of position $\vec{x}$.
- $a = V \cdot \frac{dv}{dx} = \frac{dV}{dt} = \frac{d^2x}{dt^2} = \frac{d^2s}{dt^2}$

Acceleration and deceleration motion in straight line :

1) If the velocity (V) and the acceleration (a) have the same sign (same direction) then $aV > 0$. So the motion is accelerated.
2) If the velocity (V) and the acceleration (a) have opposite sign (opposite direction) then $aV < 0$. So the motion is retarded.

5) Integration of vector functions

- If $a = \frac{dV}{dt}$ then $\int a\,dt = \int dV$ $\quad \therefore V = \int a\,dt$

  $\int_{V_o}^{V} dV = \int_0^t a\,dt$
- If $V = \frac{dx}{dt}$ then $\int V\,dt = \int dx$ $\quad \therefore x = \int V\,dt$

  $\int_{x_o}^{x} dx = \int_0^t V\,dt$
- If $a = V\frac{dV}{dx}$ then $\int a\,dx = \int V\,dV$

  $\int_{V_o}^{V} V\,dV = \int_{x_o}^{x} a\,dx$

Newton's laws

1) The momentum of a body at a moment

Is a vector quantity whose magnitude is equal to the product of the mass of this body by its velocity at this moment and its direction is the direction of the velocity itself. $\therefore \vec{H} = m\vec{V}$

2) The change of the momentum of a body $\Delta H = m(\vec{v}_2 - \vec{v}_1)$

$\Delta H = m\int_{t_1}^{t_2} a\,dt$ If the acceleration a is a function of time t

3) Newton's first law: Everybody preservers in its state of rest or of moving uniformly except in so far as it is made to change that state by an external effect,

4) Inertia principle: Everybody, as much as in it lies, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

5) Force: Newton's first law includes a definition to the force as it is the effect which changes or acts on changing the state of the body whether of rest or of moving uniformly forward in a straight line.

6) Newton's second law: The rate of change of momentum with respect to the time is proportional to the acting force and takes place in the direction in which the force is acting.

7) The equation of motion of a body whose mass is m and moves with a uniform acceleration a

$m\vec{a} = \sum \vec{F}$ where $\vec{F}$ is the resultant of the forces acting on the body

- If $a = \frac{dV}{dt}$ then the equation of motion is in the form:

$$\int_{t_1}^{t_2} F\,dt = m\int_{v_1}^{v_2} dV$$

- If $a = V\frac{dV}{dS}$ then the equation of motion is in the form:

$$\int_{s_1}^{s_2} F\,dS = m\int_{v_1}^{v_2} V\,dV$$

- If the mass is variable, then the equation of motion is in the form:

$$F = \frac{d}{dt}(mv)$$

8) The units used with the equation of motion

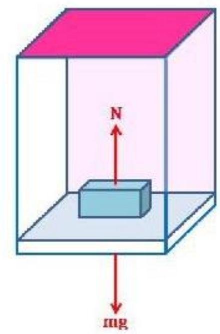
$$m(kg).\,a\left(\frac{m}{sec^2}\right) = F(newton)$$

$$m(gm).\,a\left(\frac{cm}{sec^2}\right) = F(dyne)$$

$Newton = 10^5\ dyne.$

9) Newton's third law: **To every action, there is a reaction equal in magnitude and opposite in direction.**

- **The body placed on the floor of the lift**



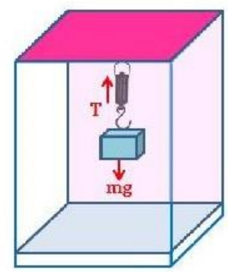
(i) **The motion is uniform** $\therefore N = mg$

(ii) **The motion by a uniform acceleration upwards**

$\therefore N = m(g + a)$

(iii) **Down wards by a uniform acc.** $\therefore N = m(g - a)$

- **A spring balance suspended from the top of a lift:**



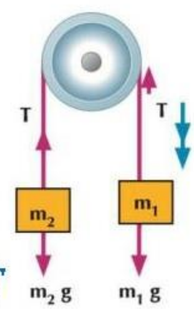
(i) **The motion is uniform** $\therefore T = mg$

(ii) **The lift is moving upwards by a uniform acceleration**

$\therefore T = m(g + a)$

(iii) **The lift is moving downwards by a uniform acceleration**

$\therefore T = m(g - a)$



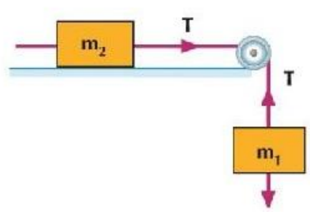
- **Equations of motion**

$m_1 a = m_1 g - T$

$m_2 a = T - m_2 g$

The pressure on the pulley $(P) = 2T$

- **Equations of motion**

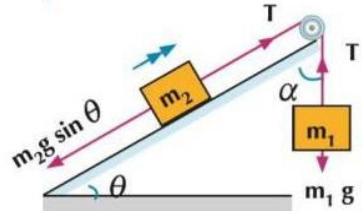


$m_{21} a = T$

$m_1 a = m_1 g - T$

The pressure on the pulley $(P) = \sqrt{2}\,T$

- **Equations of motion**



$m_2 a = T - m_2 g \sin\theta$

$m_1 a = m_1 g - T$

The pressure on the pulley $(P) = 2T\cos\left(45 - \frac{\theta}{2}\right)$

$= T\sqrt{2(1 + \sin\theta)}$

Impulse and collision

1) Impulse

If force $\vec{F}$ of a constant magnitude acts on a body during a time interval t, then the impulse of this force - denoted by the symbol $\vec{I}$ is known as the product of force vector by the time of its action i.e : $\vec{I} = \vec{F}\, t$

2) Measuring unit of impulse magnitude:

$$\textit{The measuring unit of impulse magnitude} = \textit{measuring unit of force magnitude} \times \textit{time measuring unit}$$

- In the international system of unit, the impulse magnitude is measured in $N.sec$ *unit*. It can also be measured by *any force unit in time unit*.
- **Impulse and momentum:** *impulse = change of momentum.* $\therefore F \times t = m(v_2 - v_1)$
- **The (*force − time*) graph:** The impulse can be represented by the area under the force - time graph and it can be identified by the relation: $impulse = \int_{t_1}^{t_2} F\,dt$

3) The impulsive force:

The impulse force are extremely tremendous force act for infinitesimal time interval and cause an extremely tremendous change in the momentum of the body without any acting on its position and the motion resulted at the action of these forces is called an impulsive motion. For example, the baseball when hit by the bat, the contact time between the bat and the ball is extremely infinitesimal. Although the average of the force acting on the ball is extremely tremendous. The impulse is tremendous enough to change the momentum of the ball without any change in the position of the ball.

4) Elastic collision :

If the deformation doesn't occur, heat is not generated and there is not loss in the kinetic energy. $m_1\overrightarrow{v`_1} + m_2\overrightarrow{v`_2} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}$

the sum of the two momentums after the collision directly =
the sum of the two momentums before the collision directly.

If two smooth balls collide, then the sum of their two momentums does not change due to the collision. Algebraic measures can be used as follows :

$m_1 v`_1 - m_1 v_1 = -I, \quad m_2 v`_2 - m_2 v_2 = I, \quad m_1 v`_1 + m_2 v`_2 = m_1 v_1 + m_2 v_2$

Since I is the algebraic measure to the impulse of the second ball on the first ball v_1, v_2 are the algebraic measure of the velocities before collision $v`_1, v`_2$ are the velocities after collision.

The direct collision: The two velocities before and after the collision directly are parallel to the line of the two centers at the moment of collision.

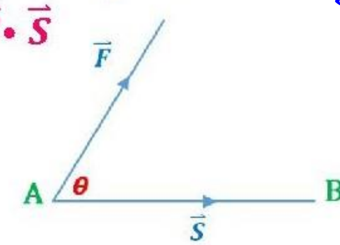
5) **Inelastic collision:**

The inelastic collision is meant that a deformation takes place, heat is generated or the bodies get contacted due to the collision process
(*loss of kinetic energy occurs*). In spite of it all, the momentum after and before collision remains as it is without change.
The momentum conservation equation is form: (in case the two bodies contacted):
$m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} = (m_1 + m_2)\vec{v}$ (by using vectors) (in case of soldering)
$m_1v_1 + m_2v_2 = (m_1 + m_2)v$ (by using algebraic measures)

Work ,Power and Energy

1) The work done by a constant force

The work done by a constant force in moving a particle from an initial position to a final position is defined as the scalar product of the force vector times the displacement vector $W = \vec{F} \bullet \vec{S}$



$\therefore W = \|\vec{F}\|\|\vec{S}\|\cos\theta$

i) If $\theta = 0 \Rightarrow W = F.S$

ii) If $\theta = 180° \Rightarrow W = -F.S$

iii) If $\theta = 90° \Rightarrow W = 0$

iv) If A, B, C are three points, then (the work done from A to B) + (the work done from B to C) = (The work done from A to C) by the action of a constant force F.

- Joule = $[Newton.meters]$

It is the magnitude of the work done by a force of magnitude one Newton in moving a body a distance one meters in the direction of the line of action of the force.

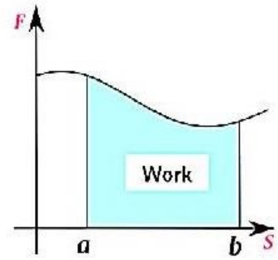
- Erg = $[dyne.cm]$

It is the magnitude of the work done by a force of magnitude one dyne in moving a body a distance one cm. in the direction of the line of action of the force. Where $Joule = 10^7\ Erg.$

- We have many other units As $Kg.wt.m.$, $gm.wt.cm.$, $gm.wt.m$ ...

2)The Work done by a variable force

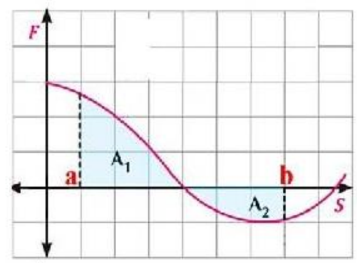
The work done by a variable force parallels the direction of the motion its magnitude F to move the body from $s = a$ to $s = b$:



$$W = \int_a^b F\,ds = \text{the area of the shaded region}$$

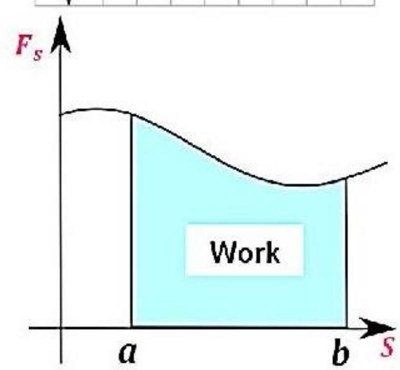
Remarks:

- In the opposite figure. If a part of the shaded area drawn above $x - axis$ and the other part drawn below $x - axis$, then :



$$W = \int_a^b F\,ds = Area\,(A_1) - Area\,(A_2)$$

- If the direction of the force inclined to $\overrightarrow{S}$ by angle θ then the component of the force in direction of $\overrightarrow{S}$ equals $F\cos\theta = F_S$, then the graphical relation between S and F_S as in the opposite figure , then



$$W = \int_a^b F_s\,ds$$

3)The kinetic energy

The kinetic energy is defined as the product of half the mass of the particle time the square of the magnitude of its velocity. $T = \frac{1}{2}mv^2$

The units of measuring kinetic energy are the same of units of measuring work.

4)The Principle of work and energy.

- If F is constant:

Let a body of mass (m) move a distance (S) under the action of the resultant of forces (F) such that its velocity changes from (v_1) to (v_2) then the work done by the resultant of these force: $W = F \times S$

$$\frac{1}{2}m({v_2}^2 - {v_1}^2) = ma \times S = F \times S = W$$

$\therefore$ The change in kinetic energy equals Work. So $T - T_o = W$

- If F is variable:

$$T = \frac{1}{2} m v^2 \qquad \therefore \frac{d}{dt}(T) = m\, v \frac{dv}{dt} = ma\, v = ma \frac{ds}{dt} = F \frac{ds}{dt}$$

$$\int_{T_o}^{T} d(T) = \int_{S_o}^{S} F\, ds \qquad \therefore \quad T - T_o = W$$

The change of the kinetic energy = work done

5) Potential energy (P)

The potential energy is the work done by acting forces on the body if it moved it from its position at this instant to a fixed position on the st. line on which motion occurs.

- Rule (1)

The change in potential energy of a particle is equal to negative the work done by the force during the motion. $P - P_o = -W$

- Rule (2)

The sum of kinetic energy and potential energy is constant during the motion. $T + P = T_o + P_o$

- Rule (3)

The potential energy of a body of mass (m)of a vertical projectile at any position of height (h) from the surface of the ground is equal (mgh)

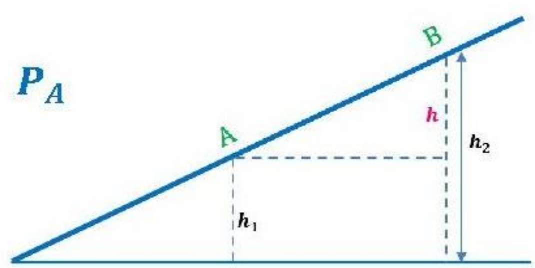
- Remark

If a body moves from point (A) to point (B) on a smooth inclined plane (for example)

The change in potential energy $= P_B - P_A$

$$= mgh_2 - mgh_1$$

$$= mg\,(h_2 - h_1) = mgh.$$



6) The Power

It is the time rate of doing work. Or It is the work done in unit time. $The\ power = \frac{dw}{dt}$

- $Power = \frac{dw}{dt}$ Since $w = \vec{F} \cdot \vec{S}$ $Power = \frac{d(\vec{F} \cdot \vec{S})}{dt}$

If the force $\vec{F}$ is constant, then $Power = \vec{F} \cdot \frac{d\vec{S}}{dt} = \vec{F} \cdot \vec{V} = FV\cos\theta$

- If $\vec{V}$ has the same direction of $\vec{F}$ then $\theta = 0$ $\therefore Power = F\,V$
- Unite of measuring power

(i) $Newton.m/sec. = watt = joule/sec.$

(ii) $Kilowatt = 1000\ watt.$

(iii) $Horse = 75\ Kg.wt.m/sec. = 735\ watt.$

- The average power:

If the force does work of magnitude w during a time interval $\Delta t = t_2 - t_1$ then: $The\ average\ power = \frac{w}{\Delta t} = \frac{w}{t_2 - t_1}$

- Using the integration to find the work

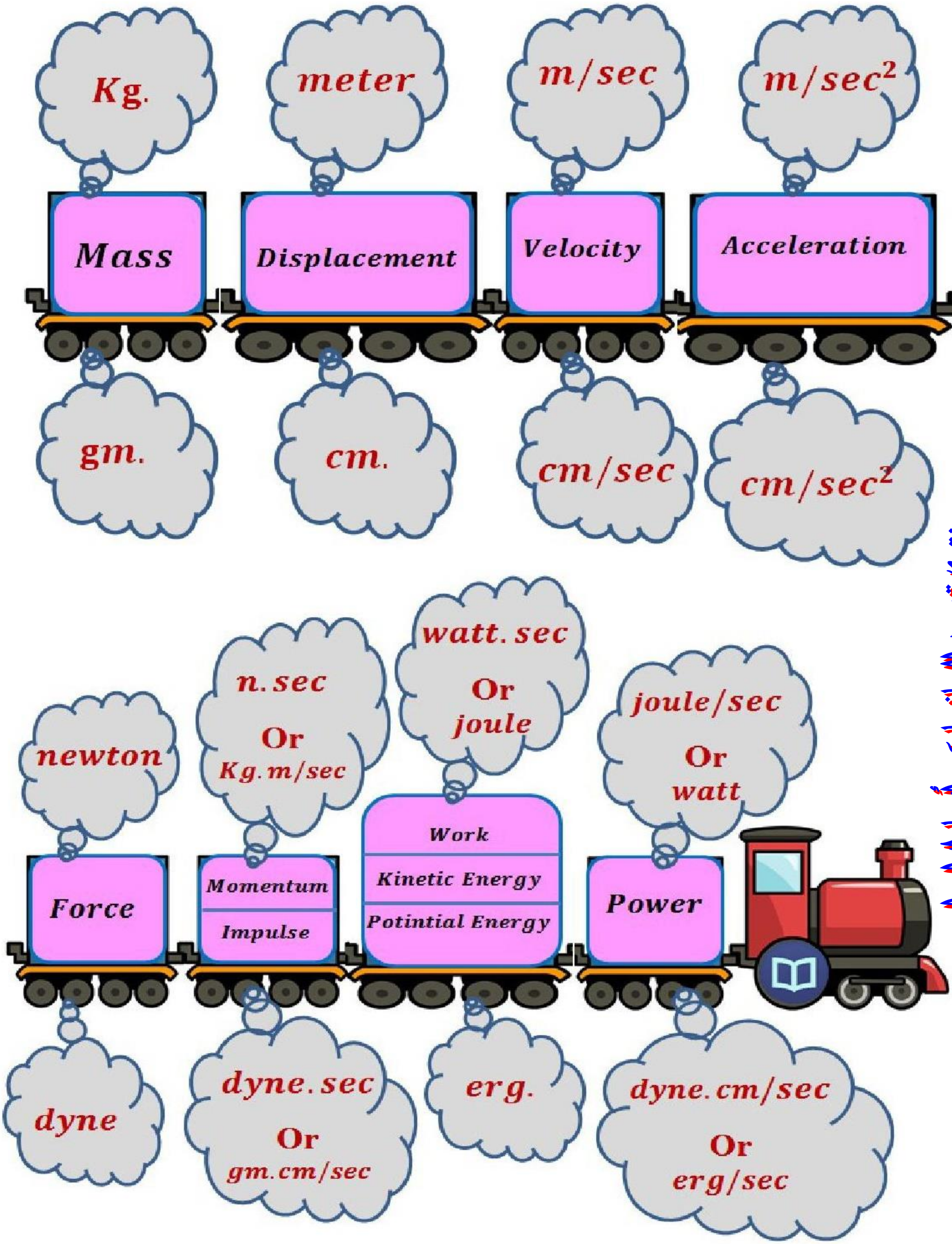
$Power = \frac{d}{dt}(w)$, $\therefore w = \int_{t_1}^{t_2}(power)dt$

- The variable power and maximum power

When the magnitude of the force F is constant , then the magnitude of power changes directly with the magnitude of the velocity V of the body and F is the arbitrary constant where $power = F\,V$

$\therefore power \propto V$ when F is constant

As long as the magnitude of the velocity changes, the magnitude of the power changes and we obtain the maximum power when the velocity is as maximum as possible . In this case, the power is called the power of machine (in general)


Kg.
Mass
gm.
meter
Displacement
cm.
m/sec
Velocity
cm/sec
m/sec²
Acceleration
cm/sec²
newton
Force
dyne
n.sec
Or
Kg.m/sec
Momentum
Impulse
dyne.sec
Or
gm.cm/sec
watt.sec
Or
joule
Work
Kinetic Energy
Potintial Energy
erg.
joule/sec
Or
watt
Power
dyne.cm/sec
Or
erg/sec